Risk sharing with the unborn

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Proefschrift

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Summary in Dutch

Financiële mee- en tegenvallers worden door pensioenfondsen doorgaans niet direct toegedeeld aan deelnemers. In plaats daarvan worden financiële schokken in eerste instantie geabsorbeerd in de dekkingsgraad van het fonds. De daardoor fluctuerende collectieve buffer van een pensioenfonds maakt het mogelijk om overschotten en tekorten gedeeltelijk door te schuiven naar toekomstige generaties. Deze dissertatie evalueert de welvaartseffecten van dergelijk beleid. Hoe aantrekkelijk is het dat risico's worden doorgeschoven naar toekomstige generaties? En hoe snel moeten pensioenfondsen herstellen van mee- en tegenvallers?

Pensioenfondsen pleiten doorgaans voor lange hersteltermijnen. Ze redeneren dat wanneer financiële schokken worden uitgesmeerd over zoveel mogelijk generaties, elke generatie slechts een klein deel van het risico draagt. De economische intutie achter deze meerwaarde van het delen van macro-economische risico's tussen niet-overlappende generaties is dat deze intergenerationele risicodeling de diversificatie van risico's verbetert; risico's worden gespreid over een groter aantal generaties dan mogelijk is in financiële markten waarin alleen overlappende generaties met elkaar kunnen handelen. Omdat financiële schokken worden uitgesmeerd over een groter aantal generaties, draagt elke generatie een kleiner deel van het risico en kan de samenleving meer risico nemen zonder dat een enkele generatie met erg veel risico geconfronteerd wordt. Het lijkt dus aantrekkelijk te zijn om financiële schokken uit te smeren over zoveel mogelijk generaties.

Dit proefschrift evalueert het effect van arbeidsmarktverstoringen op risicodeling. Het doorschuiven van risico's naar de toekomst gepaard met verstoringen in de arbeidsmarkt. Bij een financieel tekort dient een pensioenfonds te herstellen, en hanteert daarom een herstelpremie: het premieniveau is in dat geval hoog in relatie tot de waarde van opgebouwde pensioenrechten. Pensioenrechten worden

dan te duur ingekocht door werknemers, zodat er minder financiële prikkels zijn om actief deel te nemen aan de arbeidsmarkt. Werknemers zullen hierdoor sneller geneigd zijn om minder uren te werken, eerder met pensioen te gaan of zich terug te trekken uit de arbeidsmarkt. Uit empirische studies blijkt dat arbeidsmarkt keuzes inderdaad worden beïnvloed door financiële prikkels in pensioenregelingen. Vooral de keuze van de pensioenleeftijd blijkt sterk te worden bepaald door financiële prikkels.

De resultaten in deze dissertatie laten zien dat arbeidsmarktverstoringen ervoor kunnen zorgen dat het onaantrekkelijk wordt om risico's door te schuiven naar toekomstige generaties. Dit resultaat geldt ook wanneer toekomstige generaties wordt gecompenseerd voor het dragen van risico's met een substantiële beloning, de zogenaamde risico-premie. Arbeidsmarktverstoringen zorgen ervoor dat lange hersteltermijn niet langer optimaal zijn. Wanneer schokken worden uitgesmeerd over een lange herstelperiode, dan kunnen de welvaartskosten van arbeidsmarktverstoringen op den duur zeer groot worden. Immers, een lange herstelperiode betekent dat de schokken uit het verleden heel lang doorwerken. Het welvaartsverlies door een arbeidsmarktverstoring als gevolg van een nieuwe schok is hierdoor relatief groot want deze verstoring telt immers op bij de verstoringen door schokken uit het verleden. De marginale kosten van een arbeidsmarktverstoring zijn hoger als deze 'optellen' bij reeds aanwezige verstoringen als gevolg van schokken uit het verleden. Het kan daarom optimaal zijn voor een pensioenfonds om schokken op korte termijn af te rekenen met huidige deelnemers, en dus schokken niet uit te smeren over zoveel mogelijk generaties. Door schokken snel af te rekenen met de deelnemers en de fluctuaties in de collectieve buffers te beperken, herstelt een pensioenfonds de capaciteit om nieuwe schokken te absorberen zonder daarbij grote verstoringen te veroorzaken in de arbeidsmarkt.

Lange hersteltermijnen zijn met name onaantrekkelijk in de context van bedrijfstak- en ondernemingspensioenfondsen. De deelnemers van deze fondsen zijn extra gevoelig voor financiële prikkels, omdat zij zich relatief makkelijk kunnen onttrekken aan het pensioencontract door te wisselen van baan. Deze fondsen kunnen slechts in beperkte mate gebruik maken van herstelpremies, omdat deelnemers een herstelpremie relatief eenvoudig kunnen ontwijken door te veranderen van werkgever (in het geval van een ondernemingspensioenfonds), te wisselen van bedrijfstak (in het geval van een bedrijfstakpensioenfonds), of zelfstandige (bijvoorbeeld zzp'er) te worden. Het merendeel van de Nederlandse werknemers is deelnemer in één van de ruim 300 ondernemingspensioenfondsen of in één van de 73 bedrijfstakpensioenfondsen. Vanwege arbeidsmobiliteit tussen sectoren en bedrijven is het relatief moeilijk voor Nederlandse pensioenfondsen om deelnemers te committeren aan intergenerationele risicodeling, omdat werknemers hun keuzes op de arbeidsmarkt kunnen aanpassen. Een pensioenfonds op nationaal niveau, waarin werknemers uit alle sectoren en bedrijven in deelnemen, is minder gevoelig voor arbeidsmobiliteit. Een nationaal pensioenfonds is echter wel gevoeliger voor politieke risico's.

De hierboven beschreven continuiteitsrisico's zijn een belangrijke reden voor de strenge eisen Financieel Toetsingskader (FTK) van De Nederlandsche Bank (DNB). Het toezichtregime vereist dat pensioenfondsen in onderdekking binnen drie jaar uit een dekkingstekort geraken. Daarnaast dient de financiële buffer tamelijk snel te worden hersteld, namelijk binnen een periode van vijftien jaar. Het toezichtregime maakt lange hersteltermijnen dus onmogelijk. Hierdoor worden pensioenfondsen aanzienlijk beperkt in het bewerkstelligen van risicodeling met toekomstige generaties. Maar het toezichtregime is consistent met de politieke realiteit; de continuïteit van pensioenfondsen zou in gevaar komen wanneer de toezichthouder haar solvabiliteitseisen zou versoepelen.

Tevens laat dit proefschrift zien dat de welvaartswinst van risicodeling tussen generaties zeer gevoelig is voor de samenhang tussen arbeidsmarktrisico's en financiële risico's op de kapitaalmarkt. Veel studies negeren de samenhang tussen de risico's op de arbeidsmarkt en kapitaalmarkt. Maar risico's op de kapitaalmarkt staan niet op zichzelf. Een negatieve economische schok, zoals de recente kredietcrisis, zorgt niet alleen onmiddellijk voor slechte rendementen op kapitaalmarkten maar gaat tevens gepaard met een lagere economische groei en een verslechtering van de arbeidsmarkt gedurende een substantiële periode. Sterker nog: het is waarschijnlijk dat de ontwikkelingen op de kapitaalmarkt een voorafschaduwing zijn van toekomstige ontwikkelingen in de reële economie in het algemeen en de arbeidsmarkt in het bijzonder. Op de wat langere termijn beweegt de arbeidsmarkt dus mee met de kapitaalmarkt. De samenhang tussen arbeidsmarkt en kapitaalmarkt heeft belangrijke implicaties voor intergenerationele risicodeling. Het wordt namelijk minder aantrekkelijk om financiële risico's door te schuiven naar toekomstige generaties. Zij hebben immers te maken met gecorreleerde risico's op de arbeidsmarkt. Daardoor dreigen toekomstige generaties tweemaal getroffen te worden in een slecht economisch scenario: ze worden niet alleen getroffen door slechte omstandigheden op de arbeidsmarkt, maar ze moeten ook nog eens bijdragen aan tekorten die oudere generaties via de pensioenfondsen naar hen doorschuiven. Dit is een onwenselijke situatie want toekomstige generaties dragen zo te veel risico. Het kan daarom optimaal kan zijn voor een pensioenfonds om een relatief korte hersteltermijn te hanteren zodat een fonds snel herstelt van financiële schokken.

Op basis van de resultaten in dit proefschrift concludeer ik dat een lange hersteltermijnen onaantrekkelijk kan zijn voor een pensioenfonds. Een lange hersteltermijn verstoort de arbeidsmarkt wanneer werknemers het collectief dat door het fonds wordt bestreken eenvoudig kunnen ontvluchten in een flexibele arbeidsmarkt met veel arbeidsmobiliteit. Bovendien is het voor toekomstige generaties onaantrekkelijk om te delen in huidige risico's omdat ze al relatief veel risico dragen via hun menselijk kapitaal. Dit resultaat impliceert dat pensioenfondsen een minder grote risicocapaciteit hebben dan vaak gedacht, omdat het onwenselijk is om risico's door te schuiven naar de toekomst. Deze bevinding suggereert dat pensioenfondsen een relatief klein deel van het vermogen dienen te plaatsen in risicovolle investeringscategorieën, zoals aandelen, en een relatief groot deel in vastrenderende waarden. Een dergelijke prudente investeringsstrategie is consistent met een streng toezichtregime.

Summary

This dissertation contains three chapters on intergenerational risk sharing, preceded by an introductory chapter. The three chapters circulate as single-authored working papers under the same title.

The starting point of the dissertation is the inability of unborn generations to trade in current financial markets. This biological trading constraint causes financial markets to be incomplete, and thus inefficient. This point was made by Diamond (1977), Merton (1983) and Gordon and Varian (1988). More recent contributions include Smetters (2006), Bohn (2006), Cui, De Jong, and Ponds (2011), Ball and Mankiw (2007), Gollier (2008) and Gottardi and Kubler (2008). The incompleteness of financial markets implies that there can be a role for a long-lived social planner to facilitate risk sharing between non-overlapping generations. By using its financial reserves efficiently, a pension fund is able to fulfill this role. Risk sharing contracts, if properly designed, lead to an improvement in the welfare of all generations. Previous studies report large welfare gains associated with intergenerational risk sharing.

This dissertation points out that the gains from risk sharing are likely to be much smaller than often thought. I show that it can be unattractive for future generations to share in current risks. I examine two mechanisms that reduce the attractiveness of risk sharing: labor-market distortions and labor income risk. Chapters 2 and 3 analyze how distortions in the labor market erode the gains from risk sharing. Chapter 4 analyzes how the long-run labor income risk reduces the attractiveness of risk sharing.

Chapter 2 recognizes that it can be difficult to commit future generations to a risk sharing contract. Although risk sharing can be Pareto-efficient from an ex-ante perspective, generations lose from an ex-post perspective in the event that

a part of their wealth is transferred to other generations. Hence, a feasible risk sharing solution requires participation to be mandatory. Chapter 2 argues that there is a commitment problem *even* if participation is mandatory. In the situation where wealth is transferred to other generations, there is a claim on labor income which discourages labor supply. In order to reduce the burden of redistribution, workers have an incentive to reduce their labor supply, to retire early or to move to the gray or black economy. Indeed, the labor supply choices of workers are known to be quite elastic with respect to financial incentives in pension schemes, see e.g. Stock and Wise (1990), Samwick (1998) and Gruber and Wise (1999). In addition, in the case of an occupational pension fund, workers are able to evade the pension contract by switching between employers. Chapter 2 shows that the costs from distortions in labor supply and labor mobility are large, and potentially dominate the gains from risk sharing.

Chapter 2 shows that, in the presence of labor-supply distortions, it can become unattractive to shift current risk into the future. Instead, it can be optimal for a pension fund to recover from its losses in a relatively short time-period. The intuition for this result is that labor-market distortions become very large in the long run if risk is shifted into the future. The marginal costs from today's distortions are larger if they add to distortions from shocks from the past. As a result, it is optimal for a pension fund to recover from previous shocks in order to restore its capacity to absorb new shocks. In the presence of labor-supply distortions, financial gains and losses are therefore levied primarily upon the currently-living generation, preventing the pension fund from taking advantage of intergenerational risk sharing. The analysis thereby provides an economic justification for solvency regulations that require pension funds to recover from financial shortfalls in a relatively short time-period. Solvency regulations cannot be understood from the existing literature, which teaches that shocks should be smoothed over as many periods as possible, see e.g. Samuelson (1969), Merton (1969), Hall (1987), Ball and Mankiw (2007) and Gollier (2008). Short recovery periods also contrast with existing literature that finds that public policies should be set such that distortionary transfers are smoothed over time, see e.g. Barro (1979).

In addition, Chapter 2 contributes to our understanding of the role of labor-supply flexibility on optimal portfolio allocation. Labor-supply flexibility makes it more difficult for pension funds to commit future generations to share in current risk, and hence reduces risk-sharing possibilities. If risk cannot be shifted into the future, then the risk-bearing capacity of a pension fund is reduced and portfolio holdings need to be tilted towards safer assets. This result contrasts sharply with the existing literature, which finds that labor supply flexibility increases the risk appetite, see e.g. Bodie, Merton, and Samuelson (1992) and Gomes, Kotlikoff, and Viceira (2008).

Finally, chapter 2 provides a more accurate assessment of the gains from risk sharing in comparison to earlier studies. In particular, the assumption defined-contributions in Gollier (2008) is relaxed, allowing the savings rate of workers to respond to income shocks. This generalization substantially increases the risk bearing capacity, and allows the pension fund to take more advantage of the equity premium in financial markets. The gains from risk sharing more than double if the defined-contributions assumption is relaxed.

Chapter 3 shows that pension funds have the potential to mitigate distortions in the labor market. Labor-market distortions are dramatically reduced if the pension fund can apply age-differentiation with respect to relative adjustments in the value of pension rights. In the optimal pension fund policy, the value of pension rights of young workers is more responsive to economic shocks than the pension rights of old workers and retirees. Thus, downward adjustments in value of pension rights as a result of a negative economic shock (often referred to as "cuts" in pension rights) should be larger for young workers in comparison to old workers. At the same time, upward adjustments in the value of pension rights in response to a positive economic shock (often referred to as "inflation corrections" or "indexation") should also be larger for young workers. Thereby, the results in chapter 3 are an argument in favor of the introduction of age-differentiation in the policy rules of collective pension funds, as promoted in Molenaar, Munsters, and Ponds (2008).

In addition, chapter 3 compares two different approaches that are used in the risk sharing literature. Many studies on risk sharing take the perspective of a benevolent social planner who reallocates risk across generations in order to maximize the aggregated welfare of all currently-living and future cohorts. This 'social planner' approach is used for example in Gollier (2008). In the alternative approach, put forward by Teulings and De Vries (2006) and Ball and Mankiw (2007), economic agents can trade before birth in a fictitious financial market, which enables non-overlapping generations to share risk with each other, allowing for an evaluation of the gains from risk sharing. Chapter 3 shows that there is an important difference between these two approaches. The 'fictitious financial market' approach imposes that all generations are treated equal in terms of market value. A social planner, in contrast, is not constrained in this respect: many different criteria for intergenerational fairness can be applied. I show that this additional 'degree-of-freedom' of a social planner can have a large impact on ex-ante redistribution between generations. Treating cohorts equal in terms of market value implies that later-born cohorts benefit more from risk sharing than earlier-born ones. If all cohorts benefit equally from risk sharing, on the other hand, the market-value of participation in the pension fund is positive for earlier-born cohorts while being negative for later-born ones.

Chapter 4 shows that the optimal recovery period of a pension fund is crucially determined by the long-run dynamics of labor income. The numerical results in this chapter indicate that long recovery periods are no longer optimal in the presence of labor-income risk. The optimal recovery period of a pension fund is relatively short. I find that the half-life of the optimal recovery process is somewhere between 5 and 19 years, depending on the parameterization of the model. Hence, currently-living generations absorb the majority of financial shocks by themselves in the optimal risk-sharing solution, instead of shifting risk onto unborn generations.

In addition, chapter 4 shows that the gains from risk sharing are dramatically reduced once the long-run dynamics of labor income are recognized. For the benchmark parameter values, 70% of the gain from risk sharing is eroded by long-run labor-income risk. The economic intuition for this result is that comovements between stock and labor markets cause the human wealth of future generations to become correlated with financial shocks. This reduces the risk appetite of future generations and hence reduces the attractiveness of risk sharing. Interestingly, the effect of cointegration on risk sharing is large regardless of the horizon at which comovements between stock and labor markets takes place. Even if cointegration takes place at a very long horizon (i.e. if the cointegration coefficient is small) the gain from risk sharing is reduced by more than half. The intuition for this result is that the human capital of unborn generations has a long duration and hence correlates with current stock returns, regardless of whether cointegration between stock and labor markets takes place at a horizon of 5 years, 10 years or 20 years.

As the third contribution to the literature, chapter 4 generalizes the cointegration model of Benzoni, Collin-Dufresne, and Goldstein (2007) to the case where stock returns are affected by risk sources other than dividend shocks. The framework of Benzoni, Collin-Dufresne, and Goldstein (2007) assumes that dividend shocks are the single source of variation in stock returns. This assumption leads to a very strong interrelation between stock and labor markets: both dividend shocks as well as labor income shocks are closely related to future productivity levels. The assumption can therefore overstate the implications of cointegration, because variation in stock returns can also be driven by factors that are less or not related to future productivity levels, such as asset bubbles, mispricing or time-variation in discount rates. To prevent the implications of cointegration from being overstated, I introduce sources of variation in stock returns other than dividend shocks. I find that this modeling extension dramatically reduces the effect of cointegration on the portfolio holdings of individual investors. In particular, I find that the negative demand for stocks by young investors, reported in Benzoni, Collin-Dufresne, and Goldstein (2007), is not robust with respect to alternative parameter choices.

This dissertation contributes to the existing economic theory on risk sharing, which teaches that financial shocks should be smoothed over as many generations as possible. I show that shifting risk into the future is not optimal anymore once labor market distortions and the long-run dynamics of labor income are recognized. Current financial shocks should be levied primarily upon currently-living generations. Hence, the risk-bearing capacity of a pension fund is smaller than often thought, and it can be unattractive for a pension fund to have its investment portfolio tilted heavily towards risky assets. Instead, it can be optimal for pension funds to apply a prudent investment strategy in which a substantial fraction of asset holdings are invested in fixed-income products. Such a prudent investment strategy is consistent with solvency regulations that require pension funds to recover from their losses within a relatively short time-period.

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Contents

Chapter 1

Introduction

This chapter serves as an introduction to the remainder. Section 1.1 introduces the main concepts. In particular, it is motivated how risk sharing relates to the two labor-market aspects that are discussed in this dissertation: labor-market distortions and long-run labor income risk. Section 1.2 evaluates risk sharing in conjunction with these two labor-market aspects in a stylized two-agent model. Finally, section 1.3 motivates the structure of subsequent chapters.

1.1 Introduction

This section introduces the main concepts of this dissertation. Section 1.1.1 provides a general introduction to risk sharing between non-overlapping generations. Section 1.1.2 discusses show risk sharing is facilitated in the context of a pension fund. Section 1.1.3 explains why risk sharing induces distortions in the labor market. Section 1.1.4 explains how the gains from risk sharing are affected by long run labor income risk. Finally, section 1.1.5 discusses the modeling approach that is used in this dissertation.

1.1.1 Intergenerational risk-sharing

The Arrow-Debreu theory of general equilibrium teaches that the allocation of risk in financial markets will be Pareto-efficient under certain conditions. In particular, it is required that financial markets are complete. This dissertation is concerned with a deviation from Arrow-Debreu theory arising from the fact that not everyone is born at the same time. Current generations are unable to trade with the unborn generations, which causes financial markets to be incomplete and thus inefficient. This point was made by Diamond (1977), Merton (1983) and Gordon and Varian (1988). More recent contributions include Shiller (2003), Bohn (2006), Smetters (2006), Cui, De Jong, and Ponds (2011), Ball and Mankiw (2007), Gollier (2008) and Gottardi and Kubler (2008). There is thus a role for a long-lived social planner (a government of a pension fund) to reallocate risk across cohorts.

Notice that risk does not have a negative interpretation here, but is defined as chances of outcomes above or below expectations. Financial-market risk is compensated by gains in expectation, commonly referred to as the risk premium. The existence of a risk premium in financial markets creates an attractive trade-off between risk and return for investors. The objective of a social planner is therefore not to minimize risks, but to allocate risks to those best able to bear them.

If designed properly, intergenerational risk-sharing contracts lead to a welfare improvement for all generations from an *ex-ante* perspective (i.e. *before* the economic shocks materialize that determine the size and direction of risk-sharing transfers between generations). Some generations, however, may be worse off from the *ex-post* perspective (i.e. *after* the economic shocks materialize that determine the size and direction of risk-sharing transfers between generations). A pension arrangement with voluntary participation is unable to commit future generations to risk sharing, because new-born generations cannot be forced to join the risk-sharing contract if this is not in their interest from the ex-post perspective. Risk sharing between non-overlapping generations is thus not possible in a pension system in which individuals voluntarily save and invest in financial markets. Hence, a feasible risk-sharing solution requires participation to be mandatory. Risk sharing can therefore only be enforced under the government's mandate. The government has a unique "power of taxation", which enables it to make commitments on behalf of unborn generations. The government can extract rents from unborn generations in the future, implying that the future labor earnings of unborn participants can be used as collateral as when investing in financial markets.

1.1.2 Risk sharing in a pension fund

Also a pension fund with mandatory participation, acting under the government's mandate, has a "power of taxation". It is observed in practice that a drop in the value of pension fund assets leads to a rise in the contribution rate, a decline in benefit levels, or a combination of the two. By letting the contribution rate deviate from the value of pension entitlements accrued in return, the pension fund is able to extract quasi-rents from its workers: it can levy an implicit tax or provide an implicit subsidy on labor earnings. The ability to commit unborn generations to share in current risk enables a pension fund to facilitate intergenerational risk-sharing between non-overlapping generations.

The possibility to extract quasi-rents from workers allows pension funds to use future labor earnings as collateral when investing in the stock market, making it possible to outperform a laissez-faire economy. More specifically, there are two ways in which pension funds' ability to extract quasi-rents can result in a welfare improvement in comparison to a laissez-faire economy in which individuals share risk via the financial market. First, a collective pension fund is able to invest in the financial market on behalf of unborn generations, thereby alleviating the *biological constraint* of financial markets that prevents unborn individuals from trading. Second, a collective pension fund is able to alleviate the *borrowing constraint* faced by young workers, who are unable to use their future labor earnings as collateral when trading in financial markets.[1] By alleviating these two financial market constraints, a mandatory pension fund is able to outperform a laissez-faire economy and achieve higher welfare levels for its participants.

Apart from the two constraints described above, there are other ways in which pension funds are able to outperform a laissez-faire economy. For example, a pension fund may create value for its participants by providing insurance against wage-inflation risk, which may not be available in the financial market, see e.g. Cui, De Jong, and Ponds (2011). Second, pension funds can trade at low cost by taking advantage of economies of scale. Third, welfare can be substantially improved if entry costs prevent some households from investing in the stock market

[1] The borrowing constraint in financial markets is due to limited-liability in financial markets. Limited liability causes the collateral value of human capital to be restricted by the effort level chosen by participants. A large claim on labor income may provide workers with an incentive to provide a low effort level or even to default.

in a laissez-faire economy, see e.g. Abel (2001). Fourth, mandatory saving can be welfare improving if individuals suffer from myopia or other forms of bounded rationality. Finally, mandatory participation in pension funds overcomes the problem of adverse selection in insurance markets. On the other hand, the analysis also abstracts from important disadvantages of collective pension funds. For example, collective arrangements may not offer tailor-made contracts to their participants, thereby ignoring heterogeneity in preferences or personal circumstances. The analysis in this dissertation should therefore not be regarded as a complete cost-benefit analysis of collective pension funds. We solely focus on the most "fundamental" reasons for why a collective pension fund is able to outperform a solution that is based on voluntary trading in financial markets, namely the unique power-of-taxation that enables a pension fund to alleviate the *biological constraint* and the *borrowing constraint* that are faced by investors in financial markets.

Risk sharing between non-overlapping generations can also take place via wealth transfers within families. Older cohorts leaving intentional bequests to their children can help to share risk between family-members of different cohorts. The size of bequests, however, is rather small for many families. In addition, it is not possible to leave negative bequests, which constrains risk sharing possibilities. This dissertation abstracts from intra-family transfers altogether.

Risk-sharing transfers between non-overlapping generations can also take place via government policies, for example the public debt policy. If foreign investors buy domestic government bonds, then current generations are able to consume more today at the expense of consumption by future generations who have to repay the foreign debt holders at the time when the government bonds mature. [2] The risk-sharing solutions derived in this dissertation can in principle also be implemented via public debt policies. In fact, current public debt policies probably already induce various transfers between generations. If government policies already shift current risks to future generations, then there is a smaller role (or perhaps no role at all) anymore for a pension fund to do the same. This issue is beyond the scope of this dissertation.

[2] Smetters (2006) points out that an appropriate chosen tax on capital is able to substitute for public investments in the equity market. Smetters (2006) therefore concludes that the absence of a pre-funded pension scheme does not necessarily imply that there are less possibilities for risk-sharing transfers between non-overlapping generations.

More generally, it is important to stress that the first-best risk sharing solution can be achieved in various ways. There are many ways to implement the optimal risk sharing solution, by combining the instruments of pension funds, the government and the personal planning by individuals. This dissertation takes the perspective of the situation in which all risk sharing takes place via a pension fund. This is an interesting case, because pension funds play a dominant role in retirement saving in several countries. Notice however, that the analysis does not account for interactions with retirement saving via public policies and personal financial planning.

The analysis in this dissertation applies to both a nation-wide pension fund as well as to employer-based pension funds. Employer-based pension funds can be attractive because they are less vulnerable to political risk in comparison to a nation wide pension fund. In addition, public investments in private securities via a nation-wide pension fund can be controversial, as illustrated by the debates during the Clinton-administration, see e.g. White (1996), ACSS (1997), GAO (1998) and Greenspan (1999). Public investments in capital markets implies that the government effectively nationalizes a part of the economy. This can be problematic from a governance point-of-view. The decisions of the state as a shareholder of a private company may partly be driven by political interests. On the other hand, a nation-wide pension fund has the advantage that it has a democratic legitimacy, whereas employer-based pension lacks such a legitimacy. In particular, the interests of non-union workers may not be well represented by labor unions in the board of an employer-based pension fund. Furthermore, a nation-wide pension scheme enables self-employed workers to participate in risk sharing, whereas this is not possible in a pension system that is based on employer-based pension funds.

Pension funds that facilitate intergenerational risk sharing can be found in many countries. Examples include the Social Security Trust Funds in the United States, the Japan Government Pension Investment Fund, the Canada Pension Plan and the ATP funds in Denmark and the occupational pension funds in the Netherlands. Most of these funds are diversified with respect to asset class as well as internationally. Some funds, such as the US Social Security trust fund, have been put in place as a buffer against demographic shocks and are expected to deplete in the coming decades. Others, such as the Canada Pension Plan, are permanent in nature and are expected to grow in size in the coming decades. Risk sharing between non-overlapping generations is not possible in countries in which individuals save and invest for retirement on an individual retirement account. Several countries have established a funded social security tier with individual accounts, for example Australia, Ireland and Estonia.

Previous studies have reported large welfare gains associate with risk sharing in pension funds. Estimates for the welfare gain from risk sharing range from 2.3% in Cui, De Jong, and Ponds (2011) to 19.0% in Gollier (2008). In both studies, the gain from risk sharing are expressed in terms of the increase in the certainty equivalent consumption level over the full life-cycle. These studies, however, ignore two important labor-market aspects that are of importance in the context of risk sharing: labor market distortions and long-run labor income risk. These two labor-market aspects are the topic of this dissertation.

1.1.3 Labor-market distortions

Previous studies of risk sharing in pre-funded pension schemes assume lump-sum transfers between generations and ignore labor-market distortions. This is unrealistic. Real-world risk sharing contracts are not lump-sum and do distort the labor market. A drop in the value of pension fund assets raises the contribution rate, reduces benefit levels, or does both. By letting the contribution rate deviate from the value of pension entitlements, the pension fund levies an implicit tax (which may be negative) on labor earnings. Implicit taxes levied upon labor income distort the labor market. A large empirical literature finds that labor-supply choices of workers are quite elastic with respect to financial incentives in pension schemes, see e.g. the seminal contribution of Stock and Wise (1990). High contribution rates (relative to the value of accrued pension rights) provide workers with an incentive to reduce labor supply in the formal sector, by working less hours or retiring early. Hence, it is important to account for the welfare costs from labor-market distortions that are induced by risk-sharing transfers. Ignoring labor-market distortions causes the gains from risk sharing to be overestimated.

This section explains the way in which intergenerational risk-sharing is affected by labor-market distortions. Two cases are distinguished: risk-sharing between *homogenous* generations and risk-sharing between *heterogenous* generations.

First, let us consider the case of risk sharing between homogeneous generations. If each generation consists of agents that have the same characteristics, then there is no fundamental reason why the optimal risk-sharing solution should be distortionary. The pension fund is able implement the optimal risk-sharing solution without distorting the labor-market. This can be seen as follows. If all agents within a each cohort have the same characteristics, then the optimal risk exposure (and hence ex-post risk-sharing transfers) are the same for all agents within a cohort As a result, the optimal risk-sharing solution can be implemented by using non-individualized cohort-specific lump-sum transfers. Such transfers do not depend on observed behavior (labor-supply choices) and hence do not distort the labor market.

A non-distortionary implementation of risk sharing, as described above, can be infeasible in practice. There are several reasons for why such a non-distortionary implementation is not applied in real-world situations. First, limited-liability causes the size of wealth transfers to be restricted by the effort level chosen by participants. Hence, a promise to transfer wealth from future generations to current ones can be difficult to enforce, because a large claim on future labor income provides workers with an incentive to default. Second, legal constraints can prevent a pension fund from using cohort-specific lump-sum transfers. For example, some countries do not allow for cohort-specific contribution rates or cohort-specific cuts in the (relative) value of pension rights. Also political constraints can lead to a situation in which lump-sum transfers cannot be implemented. Fourth, pension fund boards may be unaware of labor-market distortions, or may have insufficient knowledge about the way policy rules can be altered in order to mitigate distortions. Finally, non-welfarist objectives of the social planner (i.e. non-welfarist notions of "fairness") can cause a lump-sum implementation to be infeasible. For example, cohort-specific cuts in the (relative) value of pension rights may be perceived as "unfair" by pension-fund boards or by pension-fund participants.

Second, let us consider the case of risk sharing between heterogeneous generations. In this situation, the optimal risk sharing solution does feature distortions in the labor-market. This can be seen as follows. In the optimal risk-sharing solution, individuals within cohorts are typically unequally affected by economic shocks in absolute terms: high-ability individuals have a higher risk-bearing capacity (in absolute terms) and therefore absorb a larger part of a shock (in absolute terms) relative to low-ability individuals. For example, if all individuals have the same degree of relative risk aversion, then the optimal exposure to economic shocks is proportional to wealth (including human wealth). Hence, the absolute size of risk sharing transfers varies among individuals within a cohort in an optimal risk-sharing solution, implying that a non-distortionary implementation of the optimal risk-sharing solution can be realized by using individual-specific lump-sum transfers.

However, individual-specific lump-sum transfers typically cannot be applied because the social planner does not observe the heterogeneity in earnings-capacities for all generations. Individual-specific lump-sum transfers are infeasible if the wealth of economic agents primarily of fully consists of future labor earnings, as is the case with young and future cohorts. The social planner does not observe the future earnings capacity of individuals, and hence cannot determine the individual-specific lump-sums. Only for older generations, for whom previous labor earnings are observed, it can be argued that the social planner has information about future earnings capacities, if one assumes that previous labor earnings are informative about the future earnings capacity. Hence, heterogeneity in earnings capacities within cohorts makes it necessary for the social planner to use distortionary taxation in the optimal risk-sharing solution.

In an ideal system of taxation, lump-sum taxes and subsidies would be related to the earnings capacity of economic agents. Tinbergens proposal of a “tax on talent” follows this argument, see Tinbergen (1970) and Tinbergen (1975). However, the government does not know enough about individuals to determine their optimal individual-specific lump sums. Instead, the government has to base taxes on observed behavior. The literature on optimal taxation that has emerged in the wake of the seminal contribution of Mirrlees (1971) starts out from the information problem which leads to distortionary taxation. This dissertation therefore assumes that a social planner lacks sufficient information about the earnings capacity of workers to determine the individual-specific taxes.

The motivation for distortions in this dissertation differs from the one that is used in the literature on optimal taxation. The literature on optimal taxation typically abstracts from capital markets or assumes that capital markets are complete (see e.g. Werning (2007)). Instead, it uses the redistributive concerns of a social planner as a motivation for distortions: the social planner is characterized by inequality-aversion, which leads to the redistribution of wealth from high- to low-skilled agents. My dissertation, on the other hand, recognizes that capital markets are incomplete (by imposing the borrowing constraint and the biological constraint) and uses the sharing of macro-economic risks as a motivation for tax distortions. Thus, this dissertation studies the optimal reallocation of macro-economic risk in an incomplete market, whereas the existing tax literature is concerned with optimal redistribution in a complete market.

For simplicity, my analysis abstracts from redistributive concerns as a motivation for tax distortions. That is: the social planner is solely concerned with risk sharing and does not care about inequality and does not engage in any ex-ante redistribution of wealth within cohorts. The distortions in this dissertation are thus fully due to risk sharing, and not due to ex-ante redistribution. Golosov, Tsyvinski, and Werning (2006) and Jacobs (2010) have shown the importance of recognizing redistributive concerns as a motivation for tax distortions. Ideally, one would like to use both risk sharing as well as redistributive concerns as motivations for distortions. This, however, is beyond the scope of this dissertation and is left for future research.

Further research will have to reveal how the welfare costs from distortions are affected if redistributive concerns are recognized as a motivation for tax distortions. Probably, the welfare costs from risk sharing *increase* once redistributive concerns are recognized as a motivation for tax distortions. The intuition here is that the tax distortions from risk sharing add to the already existing tax distortions from redistributive concerns. The marginal costs from (positive or negative) taxes from the pension fund are higher if there is already a tax in place for redistributive concerns.[3] If this is true, then the absence of redistributive concerns causes me to underestimate the welfare costs from the distortions from risk sharing.

The extent to which labor-market distortions affect risk sharing will depend on the institutional setting. In particular, a nation-wide pension fund is in a

[3] The intuition here is the result of two effects, where one dominates the other. A *positive* macro-economic shock leads to a negative tax on labor earnings by the pension fund, and thereby (partially) offsets the tax distortion from redistributive concerns and hence increases welfare. This effect, however, if dominated by the effects induced by *negative* macro-economic shocks, which lead to an additional tax on labor earnings by the pension fund, which is very costly in welfare terms because this tax adds to the already existing tax distortion from redistributive concerns. The marginal costs from a tax are much higher if there is already a tax in place, due to the non-linearity in the welfare costs from distortions.

better position to commit workers to the risk-sharing contract than an employer-based pension fund. A nation-wide pension fund can be evaded only by moving to another country or by leaving formal sector. An occupational pension fund, in contrast, can be evaded by workers more easily, namely by switching employers (in the case of an employer-based pension fund) or by switching industry (in the case of an industry-wide pension fund). In fact, rent extraction is not possible at all in a perfectly-competitive labor market: any wage-differential induced by the pension plan forces an employer to offer a compensating wage-differential to prevent an influx or outflow of workers as a result of the actuarial unfairness of the pension plan. Hence, labor-market competition implies that the employer is on the hook for a shortfall in the pension fund, not the employees.

Bohn (2010) explains that employer-based pension funds have some possibilities to extract quasi-rents from their workers. For example, it may be unattractive for workers to evade the pension policy by switching employers, due to accumulated firm-specific or industry-specific human capital. If the attractiveness of job switching is sufficiently low, an employer-based pension fund is able to extract quasi-rents from workers, and can thus facilitate intergenerational risk-sharing. Hence, private risk sharing through employer-based pension funds is feasible if specific human capital ties workers to the insurance pool of the pension fund. Also other factors can make it unattractive for workers to switch employers, such as limited portability of pension rights or implicit labor contracts involving deferred wages.

1.1.4 Long-run labor income risk

Many studies of risk sharing ignore the interrelation between capital- and labor markets. Financial market risk, however, cannot be viewed in isolation. In the long run, stock and labor markets are likely to move together, mirroring changes in the broader economy. A negative economic shock, like the recent credit crisis, not only causes an immediate drop in the value of asset prices, but is typically accompanied by a below-average growth of the economy and a below-average performance of the labor market during subsequent years. In fact, developments in financial markets are typically caused by changes in expectations with respect to future developments in the real economy in general and the labor market in particular.

In the long run, stock and labor markets therefore likely to move together, so that the factor shares of labor and capital are stationary. The long-run restriction that the factor shares of labor and capital are stationary is suggested by the form of most production functions used in macroeconomic theory. If labor and capital income are allowed to have independent trends (whether deterministic or stochastic), then the factor share of labor will approach zero asymptotically (if capital income grows faster than labor income) or the factor share of capital will approach zero (in the opposite case). This is contrary to what the data shows: although factor shares vary over time, they show no tendency to converge to zero or one.

The interrelation between stock and labor markets has important implications for risk sharing. It becomes less attractive to shift current financial market risks into the future, because the future generations now face correlated risks. More specifically, the human capital of future generations correlates with financial market risk. If current financial losses (gains) from risk-taking coincide with a decrease (increase) in the expected future wage levels, then future generations are already exposed to current risk via their human wealth. Shifting risk into the future can lead to a situation in which future generations are over-exposed to current financial market risk: in the case of a negative economic shock, future generations suffer from a bad situation on the labor market as well as a funding shortage in the pension fund. It can therefore be attractive for a pension fund to recover from financial shocks in a relatively short period of time. More generally, it will therefore be less attractive for future generations to share in current risk via a risk-sharing contract in the presence of long-run labor income risk.

1.1.5 Modeling approach

Quantitative models are used to answer the central research questions in this dissertation. Quantitative results are important, because it often needs to be determined whether a mechanism of interest plays a dominant role, or whether it is only of second-order importance. The models in this dissertation take the perspective of a small open economy that is too small to affect world prices, and hence asset prices and labor income dynamics are assumed exogenous. The assumption of exogenous factor prices greatly reduces the complexity of analytical expressions and numerical calculations. In addition, the perspective of a closed economy can be problematic in the context of risk sharing between non-overlapping generations. If goods cannot be stored, then a wealth transfer between non-overlapping generations requires the presence of other countries to lend to or borrow from. Even in the situation in which goods can be stored, the assumption of a closed economy can be restrictive, because it is not possible to store a negative amount of goods.

Throughout, the ex-ante welfare criterion is applied to evaluate the welfare of the economic agents. This welfare criterion builds on the Rawlsian approach to social justice. Rawls's thought experiment envisions a hypothetical original position before birth in which individuals agree upon a social contract behind a "veil of ignorance". In the context of risk sharing, the veil of ignorance concerns time-series uncertainty about the returns on financial assets and human wealth. That is: individuals evaluate their welfare from the perspective where the economic shocks that determine the size and direction of risk-sharing transfers between generations have not yet materialized.

1.2 Stylized two-agent setting

It is informative to start with a stylized modeling framework with two agents and two periods. The framework extends the two-agent model in Gollier (2008), which abstracts from labor-market issues. Section 1.2.1 introduces the two-agent model, which features an unborn and a currently-living agent who live in non-overlapping time-periods. Section 1.2.2 presents the autarky solution in which the two agents invest on an individual account, and are unable to share risk with each other. Section 1.2.3 treats the risk sharing solution. Sections 1.2.4 introduces distortions in labor-supply choices. Finally, section 1.2.5 explores the implications of long-run labor income risk.

1.2.1 Model

The model features two agents, where first-born agent $i = 1$ is alive during period 1 and the second-born agent $i = 2$ is alive during period 2. The periods 1 and 2 are non-overlapping, so that it is not possible for the two agents to share risks via a financial market. A long-lived social planner facilitates risk-sharing transfers

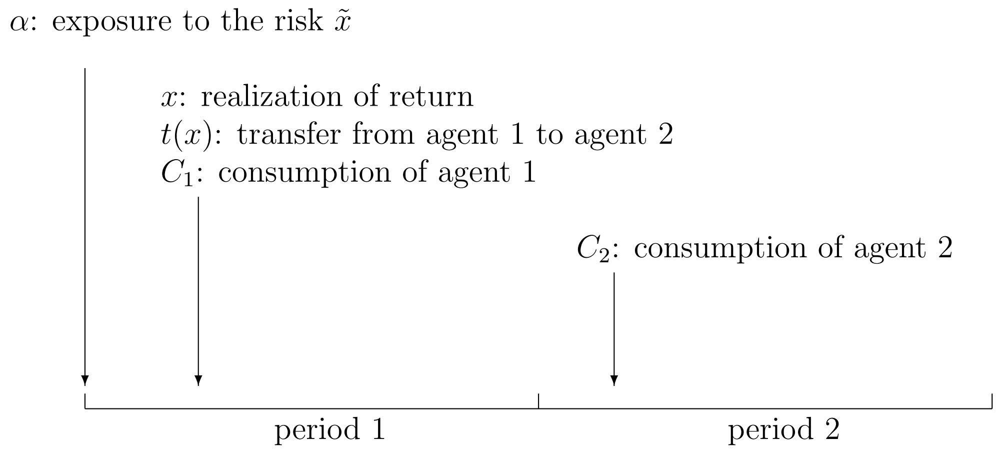


Figure 1.1: *Time-schedule of section 1.2.1.*

between the two agents. Risk sharing makes it possible for agent 2 to share in risk that materializes in period 1. Notice, however, that it is not possible for agent 1 to share in risk that realizes in period 2 since the realization occurs after agent 1 has passed away.

Throughout this dissertation, labor earnings and asset returns are assumed exogenous, consistent with the perspective of a small open economy that is too small to affect world prices. Initially, the labor earnings L_i of both agents i (i being equal to 1 or 2) are riskless: $L_i \equiv \bar{L}_i$, where $\bar{L}_i$ is a scalar. The assumption of riskless labor earnings is relaxed in section 1.2.5, where labor income risk is introduced. Initially, there is only a single source of risk in the model: stock-market risk. Given that only the stock-market risk that materializes in period 1 can be shared between the two agents, I abstract from stock investments in the second period.[4] In the first period, the financial market offers two investment opportunities: a riskless asset with zero return and a risky asset. The net return $\tilde{x}$ on the risky asset is a random variable with mean μ and variance σ^2. The

[4]This assumption is harmless when risks are small. However, risk taking in period 2 will decrease the willingness of agent 2 to share in the risks that materialize in the first period if risk exposures are high.

$\bar{L}_1$	1	labor earnings of agent 1
$\bar{L}_2$	1	labor earnings of agent 2
γ	5	coefficient of relative risk aversion
μ	$30\times0.03=0.9$	expected excess return on stocks
σ	$\sqrt{30}\times0.2=1.1$	volatility of excess return on stocks

Table 1.1: *Default parameter values in Chapter 1.*

consumption level C_1 of agent 1 consists of labor earnings plus the return on investments minus the risk-transfer from agent 1 to agent 2, while the consumption level C_2 of agent 2 equals labor earnings plus the risk transfer:

$$C_1 = \bar{L}_1 + \alpha\tilde{x} - t(\tilde{x}), \tag{1.1a}$$

$$C_2 = \bar{L}_2 + t(\tilde{x}), \tag{1.1b}$$

where α denotes the absolute amount invested in the risky asset in period 1 and where $t(\tilde{x})$ is the transfer from agent 1 to agent 2. In an open economy, the intergenerational transfer $t(\tilde{x})$ can be accomplished by lending to or borrowing from abroad. Due to the assumption of a zero risk-free rate, the risk transfer does not accumulate interest between period 1 and 2. Short-selling the risky asset (i.e. $\alpha \geq 0$) does not need to be restricted: it will follow from equations (1.6), (1.14) and (1.28) that the demand for the risky asset is positive as long as the equity premium is positive (i.e. if $\mu > 0$).

Figure 1.1 shows the time schedule for the two-agent model. The risk exposure α is determined by the social planner before the realization of the return on the risky asset occurs. Subsequently, the realization of the return determines the size of the risk-sharing transfer and the consumption levels of the agents. The risk exposure α cannot be conditioned on the realization of the return on the risky asset, which has not been realized yet at the beginning of the first period.

The two agents have identical preferences given by expected utility over consumption C_i:

$$U_i = \mathbf{E}\left[u(C_i)\right]. \tag{1.2}$$

The felicity function features constant relative risk aversion[5]

$$u(C_i) = \frac{C_i^{1-\gamma}}{1-\gamma}, \tag{1.3}$$

where γ denotes the coefficient of relative risk aversion with respect to consumption. The benchmark parameters used in chapter are contained in Table 1.1. Due to the assumption of a zero riskfree interest rate, $\bar{L}_2$ can be interpreted as the labor earnings of agent 2 discounted back to period 1. For the default parameters, the present discounted value of labor earnings of the two agents is equal. The intuition for the parameter choices for μ and σ is the following. In the situation where stock returns are independent and identically distributed (i.i.d) with a lognormal distribution, the excess mean return over an n-year period approximately equals n times the excess mean return over a 1 year period and the excess volatility over a n-year period approximately equals $\sqrt{n}$ times the excess volatility over a 1 year period. Taking the perspective of a 30-year duration of investments, and choosing the one-year expected excess return and excess volatility equal to 3% and 20% respectively, it follows that their 30-year counterparts are given by $30 \times 0.03 = 0.9$ and $\sqrt{30} \times 0.2 = 1.1$ respectively.

1.2.2 Autarky

The autarky situation corresponds the case in which there is no transfer between the two agents, i.e. $t(\tilde{x}) = 0$. The autarky solution is well-known and is repeated here for the sake of completeness. In autarky, agent 2 is not exposed to financial-market risk, i.e. $C_2 = \bar{L}_2$. Agent 1 consumes labor earnings $\bar{L}_1$ plus the proceeds from investments in the financial market. The optimal exposure α to the risk $\tilde{x}$ solves from

$$U_1 = \max_{\alpha} \left\{ \mathbf{E}\left[\frac{C_1^{1-\gamma}}{1-\gamma}\right]\right\} = \max_{\alpha} \left\{ \mathbf{E}\left[\frac{\left(\bar{L}_1 + \alpha\tilde{x}\right)^{1-\gamma}}{1-\gamma}\right]\right\}. \tag{1.4}$$

Under the assumption that the portfolio risk is small, the Arrow-Pratt approximation can be applied. Appendix 1.A shows that:

[5]The specific functional form that is assumed in equation (1.3) is used for notational convenience, but is not necessary for obtaining the results in this section. Under the Arrow-Pratt approximations that are applied in this section, all analytical expressions remain valid for any utility function $u(C_i$, with γ representing $-\frac{u''(C_i)}{u'(C_i)}$.

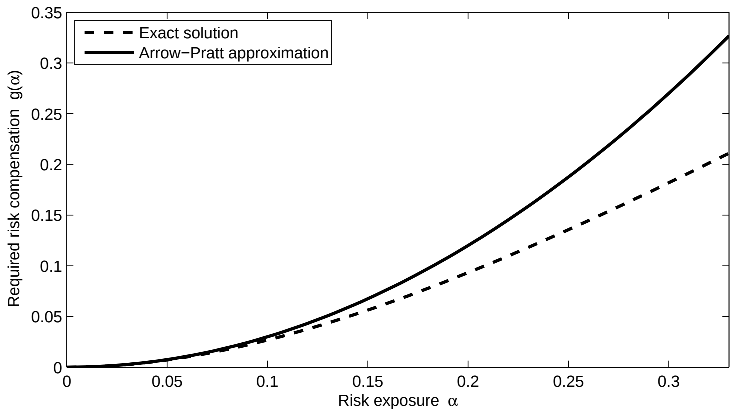


Figure 1.2: *The exact solution (dashed line) and the Arrow-Pratt approximation (solid line) for the required compensation* $g(\alpha)$ *for risk as a function of the risk exposure* α*. The calculations are based upon the default parameters contained in Table 1.1. The return* $\tilde{x}$ *on stocks is assumed to adopt a Bernoulli distribution with outcomes -0.2 and +2.0, which yields a mean of 0.9 and a volatility of 1.1, consistent with the default parameters. The exact solution* $g(\alpha)$ *is the unique solution of the equation* $\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\tilde{x}\right)^{1-\gamma}\right]-\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu-g(\alpha)\right)^{1-\gamma}=0.$ *The Arrow-Pratt approximation is given by equation* (1.5)*:* $g(\alpha)\approx\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2$.

$$\mathbf{E}\left[\frac{\left(\bar{L}_1+\alpha\tilde{x}\right)^{1-\gamma}}{1-\gamma}\right]\approx\frac{\left(\bar{L}_1+\alpha\mu-\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2\right)^{1-\gamma}}{1-\gamma}\equiv\frac{(CEQ_1)^{1-\gamma}}{1-\gamma},\tag{1.5}$$

in which CEQ_1 denotes the certainty-equivalent consumption level of agent 1, defined as the non-stochastic consumption level that yields U_1. The Arrow-Pratt approximation is based upon the first two moments (the mean and variance) of the return distribution. Samuelson (1970) provides a discussion on the limitations of mean-variance-analysis in the context of portfolio problems. In equation (1.5), the term $\alpha\mu$ represents the expected return on investments. The term $\frac{1}{2}\frac{\gamma}{\bar{L}_1}\alpha^2\sigma^2$ represents the compensation for risk required by agent 1: the agent is indifferent between paying the risk compensation on the one hand and having an exposure α to a pure risk $\tilde{x}-\mu$ on the other hand. Figure 1.2 illustrates that the Arrow-Pratt approximation is relatively accurate if the risk exposure is small, but becomes less accurate as the portfolio risk increases. The first-order derivative of equation (1.5) solves the optimal risk exposure α:

$$\alpha^{aut}=\frac{\mu}{\gamma\sigma^2}\bar{L}_1.\tag{1.6}$$

The agent has an appetite for a positive exposure to equity risk as long as the risk premium is positive ($\mu>0$) and the agent is not infinitely risk averse ($\gamma<\infty$). If the risk aversion of the agent goes to zero ($\gamma\to 0$), the agent cares only about the expected return so that the optimal risk exposure goes to infinity if $\mu>0$. For the default parameters, the agent invests $\alpha^{aut}/\bar{L}_1=0.9/(5\times 1.1^2)$=15% of wealth in the risky asset. The remaining 85% is invested in the riskfree rate.

The welfare gain that results from stock-market participation can be expressed in terms of the percentage change in the certainty-equivalent consumption level of agent 1. Substitution of equation (1.6) into equation (1.5) gives that the welfare gain from risk taking is given by:

$$\%\Delta CEQ_1=\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\text{ x100\%}.\tag{1.7}$$

Note that the expected return from risk taking is $\frac{\mu^2}{\gamma\sigma^2}\bar{L}_1$. Half of that higher expected return is offset by the cost of the attained risk. For the benchmark parameters, risk taking leads to an increase in agent 1's certainty-equivalent consumption level of $0.5\times 0.9^2/(5\times 1.1^2)$=6.8%. From this simple exercise it is inferred that the welfare gains from risk taking are large for an individual in autarky.

1.2.3 Risk sharing

A social planner is able to transfer period-1 risk to agent 2. The optimal risk-sharing solution has been treated in Gollier (2008) and is briefly summarized below for the sake of completeness. Following Gollier (2008), it is imposed that the risk transfer from agent 1 to agent 2 is characterized by a linear function

$$t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}, \tag{1.8}$$

where t_0 denotes a deterministic transfer, where α denotes the total exposure to the risk $\tilde{x}$ in period 1, and where parameter η represents the fraction of the period-1 risk exposure that is transferred to agent 2. The remaining fraction $1-\eta$ is born by agent 1.

Following Van Ewijk, Mehlkopf, and Westerhout (2011), the gain from risk sharing is measured with the *equivalent variation* associated with the risk-sharing transfer. Let the equivalent variation EQV_i be defined as the amount of wealth that agent i should be given ex-ante in the autarky case in order to obtain the same ex-ante welfare level that will be achieved by participating in the risk sharing solution. With the help of equivalent variation we can assess whether the introduction of a social planner is potentially Pareto-improving in comparison to autarky. Only if $EQV_1 + EQV_2 > 0$ can the social planner solution be potentially improving in comparison to autarky.[6] [7] Using an Arrow-Pratt approximation, the Appendix shows that aggregate equivalent variation is given by:

$$EQV_1 + EQV_2 = \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\eta^2\alpha^2\sigma^2 \tag{1.9}$$

The first term on the right-hand-side represents the expected excess return from risk taking, while the second and third term represent the compensation for risk required by agents 1 and 2 respectively. The expressions for the compensation for risk adopt the same form as in equation (1.5), but now the risk exposures of agents 1 and 2 are given by $(1-\eta)\alpha$ and $\eta\alpha$ respectively.

[6] Due to the assumption of a zero riskfree interest rate, there is no need for discounting in the aggregation of equivalent variations.

[7] Van Ewijk, Mehlkopf, and Westerhout (2011) note that this approach is similar to the Lump Sum Redistribution Authority (LRSA) as introduced by Auerbach and Kotlikoff (1987), but now applied in a stochastic environment.

An efficient risk sharing solution is derived by maximizing the aggregate equivalent variation:

$$\max_{\alpha,\eta,t_0} \{EQV_1 + EQV_2\} = \max_{\alpha,\eta,t_0} \left\{ \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\eta^2\alpha^2\sigma^2 \right\}. \quad (1.10)$$

The optimization problem specified in equation (1.10) yields the efficient frontier, i.e. the set of solutions for which there are no possibilities for a Pareto-improvement. Equivalent variation is an attractive welfare measure in the context of risk sharing, because it is unaffected by redistribution between agents (for example redistribution from rich to poor agents). Indeed, the objective function in equation (1.10) is unaffected by the deterministic transfer t_0 between agent 1 and agent 2. The parameter t_0 can be determined such that the risk-sharing solution is Pareto-improving in comparison to autarky, thereby ensuring that no agent loses from risk sharing in terms of ex-ante welfare. By requiring the risk-sharing solution to be Pareto-improving in comparison to autarky, the model rules out any welfare gains from ex-ante redistribution between the two agents (for instance between a rich and a poor agent). This property is attractive, because it ensures that the welfare gain created by the social planner can be fully attributed to gains from risk sharing. Later in this section I will derive the range for t_0 for which the risk-sharing solution is Pareto-improving in comparison to autarky.

The set of efficient solutions can alternatively be obtained by maximizing a weighted sum of utilities of agents, i.e.

$$\max_{\alpha,\eta,t_0} \{U_1 + \delta U_2\} \quad (1.11)$$

in which the social planner uses some parameter δ to weigh the relative importance of the agents. Similar to the optimization problem in equation (1.10), this objective function yields the set of Pareto-efficient solutions, and hence yields the same expressions for the optimal η and α. However, unlike equation (1.10), this objective function pins down the redistributive term t_0. It can be shown that: $t_0 = \frac{w_1 - \delta^{-\frac{1}{\gamma}} w_2}{\delta^{-\frac{1}{\gamma}}+1}$. The weighing-factor δ can be chosen such that the risk-sharing solution is Pareto-improving in comparison to autarky, thereby ruling out ex-ante redistribution between the two agents. Later in this section I will derive the range for t_0 for which the risk-sharing solution a Pareto-improvement in comparison to autarky.

The optimal decisions η^* and α^* are obtained from the first-order derivatives of equation (1.10) (or equation (1.11)). In the optimal solution, equity risk is allocated according to the relative wealth levels of the two agents:[8]

$$\eta^* = \frac{\bar{L}_2}{\bar{L}_1 + \bar{L}_2}. \tag{1.12}$$

The optimal allocation of risk in equation (1.12) ensures that the consumption of both agents is equally elastic to financial shocks:

$$\frac{\partial C_1/C_1}{\partial \tilde{x}} = \frac{\partial C_2/C_2}{\partial \tilde{x}} \approx \frac{\mu}{\gamma\sigma^2}. \tag{1.13}$$

The result in equation (1.13) is referred to as consumption smoothing: financial shocks are smoothed proportionally equally across both periods.

The optimal risk exposure solves as

$$\alpha^* = \frac{\mu}{\gamma\sigma^2}(\bar{L}_1 + \bar{L}_2) = \alpha^{aut}\left(\frac{\bar{L}_1 + \bar{L}_2}{\bar{L}_1}\right). \tag{1.14}$$

As pointed out by Gollier (2008), risk sharing increases the demand for the transferrable risk $\tilde{x}$ by a factor $\frac{\bar{L}_1+\bar{L}_2}{\bar{L}_1}$ in comparison to autarky. For the benchmark parameters the two agents have equal human wealth, implying that the demand for the risky asset doubles. The intuition for this result is that risk can be spread over a broader base, as risk sharing makes it possible for risk to be shifted towards the future, i.e. to agent 2. The optimal exposure of agent 1 to the risk $\tilde{x}$ remains unchanged in comparison to autarky: agent 1 only takes a fraction $1-\eta = \frac{\bar{L}_1}{\bar{L}_1+\bar{L}_2}$ of the total exposure which increases by a factor $\frac{\bar{L}_1+\bar{L}_2}{\bar{L}_1}$.

The risk premium is unaffected by the demand for the risky asset in this partial equilibrium setting. In a small open economy, the increase in the demand for stocks does not affect the global price of risk. In a closed economy, however, an increase in the demand for the risky asset leads to a decrease in the risk premium, thereby reducing agent 1's demand for the risky asset.

[8]Notice that the coefficient of relative risk aversion is assumed the same for both agents. If a distinction is made between γ_1 and γ_2 for agent 1 and 2 respectively, then η^* depends on the coefficients of relative risk aversion as well: $\eta^* = \frac{\gamma_1\bar{L}_2}{\gamma_2\bar{L}_1+\gamma_1\bar{L}_2}$. The share η^* of the risk allocated to agent 2 is then an increasing function of the coefficient of relative risk aversion of agent 1: $\partial\eta^*/\partial\gamma_1 > 0$.

The welfare gain from risk sharing is usually not expressed in terms of the change in utility levels U_i because such values are difficult to interpret. Instead, I follow Teulings and De Vries (2006) and Cui, De Jong, and Ponds (2011) by expressing the gain from risk sharing in terms of the change in certainty equivalent consumption levels CEQ_i. Under the Arrow-Pratt approximation, it can be shown that the change in certainty equivalent consumption of the two agents together equals the aggregate equivalent variation and is given by:

$$\Delta\left(CEQ_1 + CEQ_2\right) = EQV_1 + EQV_2 = \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}L_2 \tag{1.15}$$

Expressing the welfare gain in terms of the wealth of the unborn agent, it follows that:

$$\frac{\Delta\left(CEQ_1 + CEQ_2\right)}{\bar{L}_2} = \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\text{ x}100\%. \tag{1.16}$$

For the benchmark parameters, risk sharing results in a welfare gain of $(0.5 \times 0.9^2/(5 \times 1.1^2))$=6.7%. From this simple exercise it is inferred that the gains from risk sharing are potentially large.

The parameter t_0 governs the intergenerational fairness of the risk-sharing contract, i.e. it determines how the gain from risk sharing is divided across the two agents. If t_0 is set equal to zero, the gain from risk sharing fully accrues to agent 2, while agent 1 remains unaffected in comparison to autarky. On the other extreme, the full gain accrues to agent 1 if $t_0 = -\left(EQV_1 + EQV_2\right)$. Hence, risk sharing is Pareto-efficient in comparison to autarky from an ex-ante perspective if:

$$-\left(EQV_1 + EQV_2\right) \leq t_0 \leq 0. \tag{1.17}$$

The choice for t_0 within this interval determines the way in which the surplus from risk sharing is divided across the two agents, and depends on the criterion for intergenerational fairness that is applied by the social planner. One possible fairness criterion, put forward by Gollier (2008), imposes that all generations experience the same welfare improvement as a result of risk sharing. In the context of the two-agent setting, this criterion can be applied by imposing that both agents benefit proportionally equally from risk sharing in terms of the increase in their certainty equivalent consumption level. This is accomplished by setting:

$$t_0 = -\frac{\bar{L}_2}{\bar{L}_1 + \bar{L}_2}\left(EQV_1 + EQV_2\right), \tag{1.18}$$

where $\bar{L}_2/(\bar{L}_1 + \bar{L}_2)$ denotes the relative wealth of agent 2. For the benchmark parameters, this fairness-criterion implies that both agents gain 3.4% in terms of certainty equivalent consumption.

A second criterion for intergenerational fairness, suggested by Teulings and De Vries (2006) and Ball and Mankiw (2007), imposes that all generations are treated equal in terms of market value. Risk sharing transfers are a zero-sum game in market value, implying that equal treatment in market value implies that the value of the risk sharing transfer between agents is equal to zero. In the context of the two-agent setting, this criterion implies that the market value of the risk sharing transfer is equal to zero, i.e. [9]

$$t_0 = 0 \tag{1.19}$$

Imposing fairness in terms of market value results in the situation in which the current agent invests according to the autarky solution and in which the unborn agent is able to trade in the financial market before birth (i.e. in period 1). Indeed, this is the situation is examined by Teulings and De Vries (2006). Equality in market terms implies that the gain from risk sharing fully accrues to agent 2. For the benchmark parameters it implies that agent 2 gains 6.8% in terms of certainty equivalent consumption, whereas agent 1 gains nothing.

Applying market value as a criterion for intergenerational fairness in a *partial* equilibrium framework raises questions. Arguably, it would be better to apply a *general* equilibrium framework in which current generations can trade with future generations and market prices would adjust. Such 'fictional' trading between non-overlapping generations would result in an adjustment of the price of risk, and hence affect redistribution between agents. Possibly, the adjustment in the price of risk would result in a situation in which agent 2 does not accrue the full gain from risk sharing.

Nevertheless, market value is attractive as a criterion for intergenerational fairness. Market value does not require assumptions with regards to the preferences

[9] Recall that the variable $\tilde{x}$ represents the excess return on stocks, i.e. the return on stocks in excess of the return on the riskfree rate. Hence, any multiple of the return $\tilde{x}$ can be obtained in the financial market at zero costs by buying stocks with money that is borrowed against the riskfree rate. The transfer $t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}$ thus has a market value of zero if t_0 is set equal to zero.

of individuals (in contrast to utility value). Furthermore, the property of market value that it gives a full compensation for risk to future cohorts may in fact be a rather attractive property. In real-world applications, the interests of unborn generations may not be well represented by currently-living ones. Market value as a criterion for fairness can therefore be a powerful instrument to prevent currently-living generations from shifting risk onto unborn generations without providing a proper risk compensation.

From the simple exercise in this section it is inferred that these two fairness-criteria differ substantially from each other. In particular, treating cohorts equal in welfare terms implies that the market value of the risk sharing contract is positive for the currently-living agent while being negative for the unborn agent. Treating cohorts equal in market terms implies that the unborn agent benefits from risk sharing, while the currently-living agent does not. This issue is examined in a richer modeling environment in subsequent chapters.

It can also be inferred from this section that a risk sharing model does not require a 'social planner'. The gains from risk sharing can also be obtained by allowing economic agents to trade in a 'fictitious financial market' before birth. In the context of the two-agent setting, this approach corresponds to the in case where $t_0 = 0$, in which case agent 2 trades in the financial market before birth (i.e. in period 1). Hence, there are two approaches for the assessment of risk sharing: the 'social-planner' approach and the 'fictitious-financial-market' approach. It can be inferred from this two-agent model that there is an important difference between the two approaches. The 'fictitious-financial-market' approach imposes equality in market terms (i.e. $t_0 = 0$), whereas a social planner is flexible in terms of the choice for the criterion for intergenerational fairness. As a result, the 'social planner' approach has an additional degree of freedom. This issue is examined in a richer modeling environment in subsequent chapters.

1.2.4 Endogenous labor supply

The previous section assumed that the risk-sharing transfer $t(\tilde{x})$ takes the form of a lump-sum transfer, i.e. a transfer that is unrelated to labor earnings. As explained in section 1.1.3, a non-distortionary implementation of risk sharing is not possible, due to unobserved heterogeneity within cohorts.

To keep the analysis simple, the model does not feature heterogeneity within cohorts. However, the reader should bear in mind that unobserved heterogeneity within generations is the reason for why individual lump-sum transfers cannot be applied, as explained in section 1.1.3.

To evaluate how the gains from risk sharing are affected by labor market distortions, let us consider the following extension of the two-agent model. Let us assume that the wealth $\bar{L}_1$ of agent 1 consists of tangible assets, while the wealth of agent 2 solely consists of future labor earnings. For example, one can think of agent 1 as being an old worker with a large amount of accumulated pension rights (which can be cut in the event of a bad economic outcome) and agent 2 as being an unborn individual whose wealth fully consists of future labor earnings. Therefore it is assumed that the social planner is able to apply lump-sum transfers to agent 1, but is required to apply distortionary taxes and subsidies to agent 2. The intuition here is that individual-specific lump-sum transfers are infeasible if the wealth of economic agents primarily of fully consists of future labor earnings, as is the case with young and future cohorts. The social planner does not observe the future earnings capacity of individuals, and hence cannot determine the individual-specific lump-sums. Only for currently-living generations for whom previous labor earnings are observed, it can be argued that the social planner has information about future earnings capacities, if one assumes that previous labor earnings are informative about the future earnings capacity.

Let the labor earnings of agent 2 be redefined as the product of the wage rate w_2 and the labor supply level h_2 of agent 2, i.e. $\tilde{L}_2 \equiv w_2 h_2$. This section abstracts from labor income risk by assuming the wage rate w_2 to be constant. Labor income risk will be introduced in section 1.2.5. The labor-supply choice h_2 of agent 2 is assumed to be elastic with respect to taxes and subsidies that result from the risk sharing transfer $t(\tilde{x})$ in a way that will be specified later in this section. The labor earnings of agent 1 remain undistorted and are constant at level $\bar{L}_1$. Figure 1.3 shows the time schedule for the model. The Figure illustrates that labor supply choice h_2 of agent 2 can be conditioned upon the realization of the risk sharing transfer.

The preferences of agent 2, previously given by equation (1.2), are now specified over consumption C_2 and labor h_2:

$$U_2 = \mathbf{E}\left[u(C_2, h_2)\right], \tag{1.20}$$

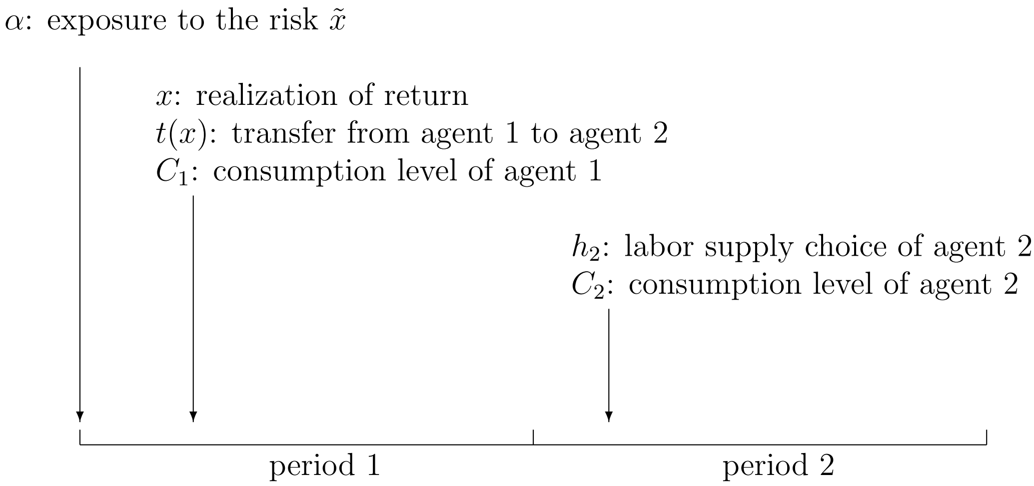


Figure 1.3: *Time-schedule of section 1.2.4.*

where the felicity function, previously given by equation 1.3, alters into:

$$u(C_2, h_2) = \frac{1}{1-\gamma}\left(C_2 - \frac{\epsilon}{\epsilon+1}\left(h_2\right)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}\left(h_2^*\right)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}. \tag{1.21}$$

In equation (1.21), parameter ϵ represents the elasticity of labor supply with respect to the marginal wage rate. Accordingly, a drop in the wage rate by one percent results in a decline in the labor supply level of ϵ percent. Originating from Greenwood, Hercowitz, and Huffman (1988), the specification in equation (1.21) features no income effects in labor supply. Labor-supply decisions are determined solely by the effective marginal wage rate against which labor is supplied. Income effects in labor supply are found to be small when compared to substitution effects, see e.g. Blundell and MaCurdy (1999). In any case, the complexity of the analysis is dramatically reduced if income effects are absent. Notice that the absence of income effects implies that ϵ has the interpretation of the *compensated* elasticity of labor supply, as it measures the substitution-elasticity of labor supply. If labor supply is undistorted, labor supply choices are given by:

$$h_2^* = w_2^\epsilon. \tag{1.22}$$

The inclusion of the term $\frac{\epsilon}{\epsilon+1}\left(h_2^*\right)^{\frac{\epsilon+1}{\epsilon}}$ in the preference specification of equation

(1.21) has two attractive implications. First, preferences simplify into standard CRRA utility over consumption C_2 if labor supply is undistorted. The model thus reduces into section 1.2.3 if distortions in labor supply are absent. Second, the relative risk aversion with respect to consumption C_2 of agent 2 will be close to γ if labor supply levels are not too far away from the first-best level h_2^*, regardless of the parameter of labor-supply elasticity ϵ. This property allows for a sensitivity analysis for labor supply elasticity ϵ under approximate *ceteris paribus* conditions with respect to coefficient of relative risk aversion γ.

Similar to the previous section, the risk-sharing transfer is linear: $t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}$. Under elastic labor supply, however, the deterministic component t_0 matters for the social surplus from risk sharing. For simplicity, assume that $t_0 = 0$. Similar to section 1.2.3, setting $t_0 = 0$ results in a risk-sharing solution that treats both agents fair in terms of market-value.

Furthermore, it is assumed that the risk-sharing transfer $t(\tilde{x})$ is levied upon the labor earnings of agent 2 in the form of *proportional* taxes and subsidies. Proportional taxes and subsidies are, by approximation, optimal if preferences over consumption obey constant relative risk aversion. After all, it is optimal for agents to share proportionally equally in a shocks if all agents have the same level of relative risk aversion. This result holds exactly in the absence of distortions, and holds by approximation in the model with elastic labor supply if distortions are small. In the presence of distortions, linear taxes are not optimal anymore.

The assumption of linear taxes causes me to overstate the welfare costs from distortions. The extent to which the welfare costs from distortions are overstated by the model depends on two factors: (1) the elasticity of labor supply and (2) the degree of heterogeneity within cohorts (which is not explicitly modeled to keep the analysis simple). In the extreme case where the elasticity of labor supply goes to infinity or in which there is no heterogeneity, it is optimal for the social planner to apply lump-sum transfers that do not depend on labor earnings. In the other extreme case, where the elasticity of labor supply goes to zero or in which heterogeneity becomes infinite, linear taxation is optimal.

In addition, notice that in the context of a nation-wide pension scheme, the assumption of linear taxes abstracts from the complication that individuals with a very low ability typically receive a certain social minimum income at all times, and hence do not participate in risk sharing.

Anyway, proportional transfers imply that the marginal tax/subsidy on labor earnings equals the average tax/subsidy and is given by $t(\tilde{x})/w_2h_2$. As a result, the effective marginal wage rate against which labor is supplied by agent 2 equals $w_2(1+t(\tilde{x})/(w_2h_2))$. It follows from equation (1.22) that the labor-supply choice h_2 of agent 2 is given by:

$$h_2 = \left(w_2\left(1+\frac{t(\tilde{x})}{w_2h_2}\right)\right)^{\epsilon} = h_2^*\left(1+\frac{t(\tilde{x})}{w_2h_2}\right)^{\epsilon}. \tag{1.23}$$

In the special case where labor supply is inelastic, i.e. $\epsilon = 0$, labor supply choices reduce into equation (1.22). Under elastic labor supply, the labor-supply choice h_2 of agent 2 is a random variable as it depends on the stochastic return $\tilde{x}$. The labor supply choice h_2 appears on both sides of equation (1.23) and does not adopt an explicit solution. Explicit expressions in this section are therefore derived on the basis of the following approximation of the labor-supply choice:

$$h_2 = h_2^*\left(1+\frac{t(\tilde{x})}{w_2h_2}\right)^{\epsilon} = h_2^*\left(1+\frac{\eta\alpha\tilde{x}}{w_2h_2}\right)^{\epsilon} \approx h_2^*\left(1+\frac{\eta\alpha(\tilde{x}-\mu)}{w_2h_2^*}\right)^{\epsilon}. \tag{1.24}$$

Using the approximation for labor-supply choices in equation (1.24), Appendix 1.A derives that the expected utility of agent 2 adopts the following approximation:

$$\begin{aligned}
\mathbf{E}\left[u(C_2,h_2)\right] &= \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2h_2+\eta\alpha\tilde{x}-\frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}}+\frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \\
&\approx \frac{1}{1-\gamma}\left(w_2h_2^*+\eta\alpha\mu-\frac{1}{2}\frac{\gamma}{w_2h_2^*}\eta^2\alpha^2\sigma^2-\frac{1}{2}\frac{\epsilon}{w_2h_2^*}\eta^2\alpha^2\sigma^2\right)^{1-\gamma} \\
&\equiv \frac{1}{1-\gamma}\left(CEQ_2\right)^{1-\gamma}
\end{aligned} \tag{1.25}$$

Similar to equation (1.5), equation (1.25) is a Taylor approximation that is based upon the first two moments of the return distribution: the mean and the variance. The term $\frac{1}{2}\frac{\gamma}{w_2h_2^*}\eta^2\alpha^2\sigma^2$ represents the compensation for risk and has been discussed in the previous section. The term $\frac{1}{2}\frac{\epsilon}{w_2h_2^*}\eta^2\alpha^2\sigma^2$ is due to elastic labor supply and represents the welfare loss that results from the labor-supply distortions that are induced by the risk-sharing transfer. The welfare loss from distortions is measured with error, as illustrated in Figure 1.4. The approximation in equation (1.25) is accurate for low values of ϵ but becomes less accurate if labor supply elasticity is high. In the exercise of Figure 1.25, the welfare costs $f(\epsilon)$ are a concave function of ϵ. This result, however, does not hold in general. For other parameter choices or other return distributions, $f(\epsilon)$ is a convex function of ϵ.

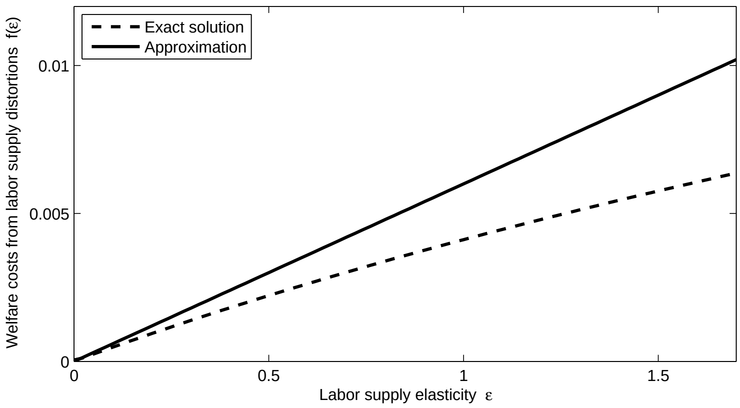


Figure 1.4: *The exact solution (dashed line) and the approximation (solid line) for the welfare loss (as a fraction of undistorted labor earnings $w_2 h_2^* = 1$). The labor supply choices h_2 obey equation (1.23), where the risk exposure is set equal to $\eta\alpha = 0.1$. The calculations are based upon the default parameters contained in Table 1.1. The return $\tilde{x}$ on stocks is assumed to adopt a Bernoulli distribution with outcomes -0.2 and +2.0, which yields a mean of 0.9 and a volatility of 1.1, consistent with the default parameters. The exact welfare loss $f(\epsilon)$ from distortions in labor supply is calculated as $\mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2 h_2 + \eta\alpha\tilde{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] - \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2 h_2^* + \eta\alpha\tilde{x} - f(\epsilon)\right)^{1-\gamma}\right] = 0$, where h_2 is given by equation (1.23). The Arrow-Pratt approximation for the welfare loss is given by equation (1.25): $f(\epsilon) \approx \frac{1}{2}\frac{\epsilon}{L_1^*}\eta^2\alpha^2\sigma^2$.*

The result in equation (1.25) implies that the optimization problem of the social planner, previously given by equation (1.10), now alters into:

$$\max_{\alpha,\eta}\left\{\alpha\mu-\frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2-\frac{1}{2}\frac{\gamma+\epsilon}{w_2h_2^*}\eta^2\alpha^2\sigma^2\right\}. \tag{1.26}$$

In contrast to the case with inelastic labor supply, equity risk is not shared according to relative wealth levels. The presence of labor-supply distortions makes it less attractive for agent 2 to bear risk:

$$\eta^*=\frac{\frac{\gamma}{\gamma+\epsilon}w_2h_2^*}{\bar{L}_1+\frac{\gamma}{\gamma+\epsilon}w_2h_2^*}. \tag{1.27}$$

The optimal equity exposure is given by

$$\alpha^*=\alpha^{aut}\left(\frac{\bar{L}_1+\frac{\gamma}{\gamma+\epsilon}w_2h_2^*}{\bar{L}_1}\right), \tag{1.28}$$

and is decreasing in the elasticity of labor supply ϵ. Hence, elastic labor supply reduces the demand for the risky asset. This result is driven by two effects. First, risk sharing transfers are accompanied by distortions in labor supply choices, which reduce welfare. Second, risk sharing causes the labor-supply behavior of agent 2 to become more pro-cyclical, thereby having a destabilizing effect on consumption levels. These two effects lead to the situation in which labor-supply flexibility (i.e. the ability of individuals to vary labor supply ex-post) reduces the risk-bearing capacity. This finding stands in striking contrast to the analysis of Bodie, Merton, and Samuelson (1992), who find the opposite effect. The result in their paper is due to income effects in labor supply: a positive (negative) income shock decreases (increases) the marginal utility from working, causing labor supply to become more pro-cyclical. In my analysis, the gains and losses from risk taking are levied (in the case of agent 2) in the form of taxes and subsidies. Taxes and subsidies not only induce income effects in labor supply, but also substitution effects. Substitution effects work in the opposite direction as income effects, and thus have an opposite effect on the risk-bearing capacity: labor supply behavior becomes destabilizing instead of stabilizing.

The elasticity of consumption with respect to financial shocks is given by:

$$\frac{\partial C_1/C_1}{\partial\tilde{x}}\approx\frac{\mu}{\gamma\sigma^2}>\frac{\mu}{\gamma\sigma^2}\,\frac{\gamma}{\gamma+\epsilon}\approx\frac{\partial C_2/C_2}{\partial\tilde{x}}. \tag{1.29}$$

Equation (1.29) points out that labor supply effects introduce a trade-off between consumption smoothing on the one hand and minimizing distortions in the labor market on the other hand. Labor market distortions are avoided if shocks are not transferred to agent 2, i.e. $\frac{\partial C_2/C_2}{\partial \tilde{x}} = 0$, in which case risk sharing is absent. On the other extreme, consumption smoothing is accomplished if both agents share proportionally equally in shocks, i.e. $\frac{\partial C_2/C_2}{\partial \tilde{x}} = \frac{\partial C_1/C_1}{\partial \tilde{x}}$, in which case labor-supply distortions are relatively large. The optimal solution lies inbetween these two extremes. The gain from risk sharing, previously given by equation (1.16), alters into:

$$\frac{\Delta\left(CEQ_1 + CEQ_2\right)}{\bar{L}_2} = \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}\frac{\gamma}{\gamma+\epsilon}\text{ x100\%.} \tag{1.30}$$

It follows that the fraction of welfare gain from risk sharing that is eroded by distortions is given by:

$$\frac{CEQ|_{\epsilon=0} - CEQ}{CEQ|_{\epsilon=0} - CEQ^{aut}} = \frac{\epsilon}{\gamma+\epsilon}. \tag{1.31}$$

where $CEQ \equiv CEQ_1 + CEQ_2$. Thus, the welfare gain from risk sharing is fully preserved if labor supply is inelastic ($\epsilon = 0$) while it is fully eroded if labor supply is infinitely elastic ($\epsilon \to \infty$). For the default parameter $\gamma = 5$, labor-market distortions erode 4.8%, 9.1% and 16.7% of the gain from risk sharing if the elasticity of labor supply ϵ equals 0.25, 0.5 and 1 respectively. Labor-supply distortions are more costly for low levels of the parameter of relative risk aversion γ since these coincide with high levels of risk taking, and thus larger risk transfers between the two agents. For example, if $\epsilon = 0.5$, labor-market distortions erode 4.8%, 9.1% and 20% of the gain from risk sharing if the coefficient of relative risk aversion equals 10, 5 and 2 respectively.

The analytical expressions in this section provide us with useful qualitative results. We have learned that labor-supply effects reduce the demand for the risky asset and reduces the attractiveness of shifting risk into the future. Quantitatively, however, the expressions in this section are of limited relevance. Figure 1.5 shows that the accuracy of equation (1.31) is weak. In addition, Figure 1.6 illustrates that the welfare effects of elastic labor supply are quite sensitive to the parameters μ and σ of return distribution, contrary to what the expression in equation (1.31) suggests.

The simple analysis in this section points out that the gains from labor market distortions are subject to labor-supply distortions. The welfare losses reported in

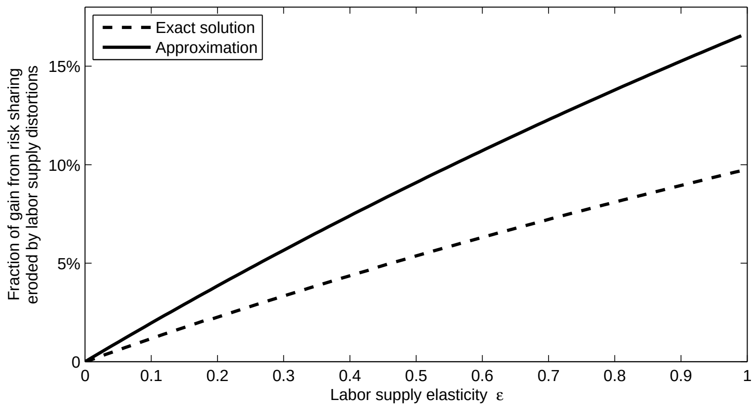
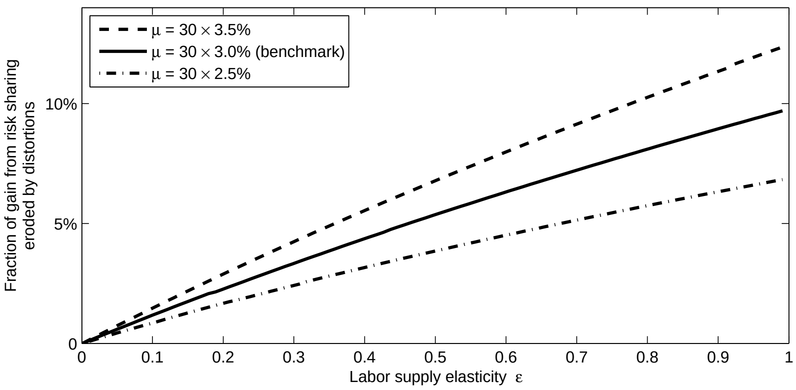


Figure 1.5: *The exact solution and the Arrow-Pratt approximation for the fraction of the social surplus from risk sharing that is eroded by labor-supply distortions. Calculations are on the basis of the default parameters. As in previous figures,* $\tilde{x}$ *adopts a Bernoulli distribution with realizations -0.65 and +1.55, both with probability 0.5, so that* $\mu = 0.9$ *and* $\sigma = 1.1$.

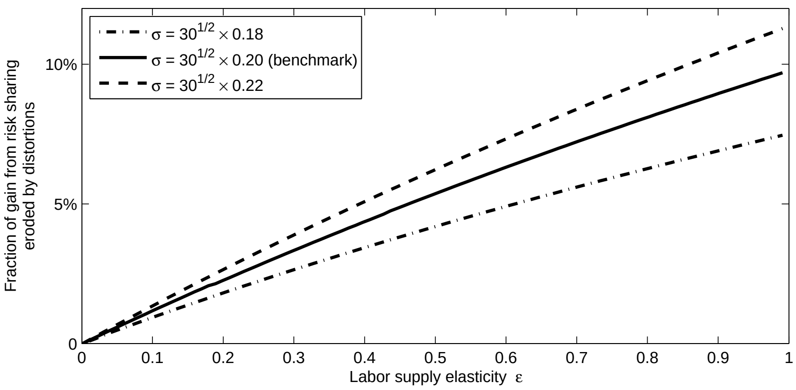
this stylized analysis are relatively modest. Subsequent chapters, however, show that labor-market distortions can be substantially more costly in a richer modeling environment, which better captures the workings of real-world pension policies.

Notice that the distortions from risk sharing are evaluated in isolation from other distortions in the labor market. However, the distortions from the pension fund interact with the distortions induced by the taxation of labor for public spending. The welfare costs from distortions are therefore underestimated, because the distortions add to already existing distortions in the labor market. If the welfare costs from distortions are non-linear, then the costs from distortions from risk sharing become larger if other distortions are added to the model.[10]

[10]Notice that a negative tax (i.e. a subsidy) on labor earnings from the pension fund in the event of a surplus reduces the burden of taxation on labor from other government taxes, thereby reducing distortions in the labor market and hence increasing welfare. On the other hand, in the event of a shortfall a pension fund induces a positive tax on labor earnings, which add to the already existing burden of taxation on labor from other government taxes. Assuming that the welfare costs from distortions are non-linear, it follows that the welfare losses associated with shortfalls dominate the welfare gains associated with surpluses.



(a) Sensitivity w.r.t. μ



(b) Sensitivity w.r.t. σ

Figure 1.6: *Sensitivity of the result in Figure 1.5 with respect to μ and σ.*

1.2.5 Long-run labor income risk

Previous sections assumed the wage rate to be riskless. This section explores how the gains from risk sharing are affected by long-run labor income risk. For simplicity, let us abstract from elastic labor supply by extending the analysis in section 1.2.3. The labor earnings of agent 2 become stochastic and are therefore denoted by $\tilde{L}_2$ instead of $\bar{L}_2$. For simplicity, the labor earnings of agent 1 remain constant at level $\bar{L}_1$.

I take the perspective where the period-2 labor earnings $\tilde{L}_2$ and period-2 dividend levels $\tilde{D}_2$ are subject to a common risk factor $\tilde{x}$:

$$\tilde{L}_2 = \bar{L}_2 \left(1 + k\tilde{x}\right), \tag{1.32}$$

$$\tilde{D}_2 = \bar{D}_2 \left(1 + \tilde{x}\right), \tag{1.33}$$

where $\bar{L}_2$, $\bar{D}_2$ and k are constants and where $\tilde{x}$ is a random variable with mean μ and variance σ^2. The risk factor $\tilde{x}$ materializes during period 1, and can be interpreted as information on period-2 dividends and labor earnings. Let the period-1 stock price be defined as the discounted value of period-2 dividends. Abstracting from time-variation in discount rates, it follows from equation (1.33) that the period-1 stock return is fully driven by period-2 dividend shocks and is given by $\tilde{x}$.[11] [12] Similarly, it follows from equation (1.32) that the period-1 return on the human wealth of agent 2 is equal to $k\tilde{x}$. Parameter k measures the exposure of period-2 labor earnings to period-1 stock return variation. In the special case where $k = 0$, the labor earnings of agent 2 are constant and the model reduces into section 1.2.3.

Note that the economic shock $\tilde{x}$ affects stock returns directly (in period 1) and labor earnings with a lag (not until period 2). The underlying intuition for this

[11] With the period-1 stock price defined as the discounted value of period-2 dividends, it holds that the variation in stock returns is driven by two risk factors: shocks to period-2 dividend levels and shocks in the discount rate. This simple exercise abstracts from time-variation in discount rates, and it follows that all the variation in stock returns is driven by dividend shocks. I will return to this issue later in this section.

[12] Period-2 dividends are stochastic and should therefore be discounted by using a stochastic discount factor. This technique will be applied in subsequent chapters. In this exercise, however, I simply use the riskfree rate (which is assumed equal to zero) to discount period-2 dividends back to period 1.

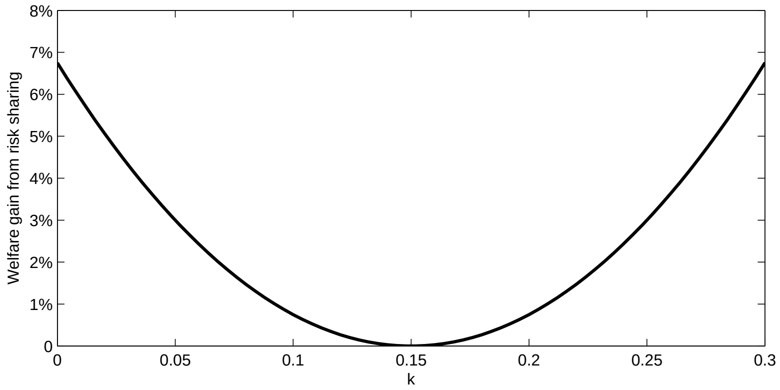


Figure 1.7: *The welfare gain from risk sharing as a function of k. Calculations are based upon the default parameters.*

modeling approach is that period-1 stock prices represent the discounted value of period-2 dividends, and are thus subject to period-2 productivity levels. Similarly, the period-1 return on the human wealth (i.e. the discounted value of future labor earnings) of agent 2 is also affected directly in period 1 by the shock $\tilde{x}$. An alternative reason for why labor earnings are slow to respond to economic shocks is that wages are inelastic at short horizons due to wage rigidity. The optimization problem of the social planner, previously given by equation (1.10), alters into:

$$\max_{\alpha,\eta}\left\{\bar{L}_1+\bar{L}_2+\alpha\mu-\frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2-\frac{1}{2}\frac{\gamma}{\bar{L}_2}\left(\eta\alpha+k\bar{L}_2\right)^2\sigma^2\right\}. \tag{1.34}$$

In equation (1.34), agent 2's exposure to the transferrable risk $\tilde{x}$ equals $\eta\alpha+k\bar{L}_2$, where $k\bar{L}_2$ represents the exposure via human wealth and where $\eta\alpha$ is the exposure through the risk-sharing transfer. The demand for the risky asset solves as:

$$\alpha^*=\frac{\mu}{\gamma\sigma^2}\left(\bar{L}_1+\bar{L}_2\right)-k\bar{L}_2. \tag{1.35}$$

Comparing equations (1.14) and (1.35), it follows that labor income risk reduces the demand for the risky asset by an amount $k\bar{L}_2$, which is the risk exposure that agent 2 already has to period-1 stock market risk via human wealth. Autarky is Pareto-efficient if

$$k=\mu/(\gamma\sigma^2). \tag{1.36}$$

In this knife-edge case, it holds that agent 2's exposure $k\bar{L}_2$ to the transferable risk $\tilde{x}$ via human wealth equals the optimal exposure $(\mu/(\gamma\sigma^2))\bar{L}_2$. Figure 1.7

illustrates the welfare gain from risk sharing as a function of k for the default parameters. If $k = 0$, labor earnings are riskless and the gain from risk sharing equals 6.7%, as in section 1.2.3. Autarky is Pareto-efficient if $k = 0.9/(5 \times 1.1^2) = 0.15$, in which case there is no role for the social planner (i.e. $\eta^* = 0$). For values of k smaller than this knife-edge case, the social planner facilitates a positive risk exposure from agent 1 to agent 2 (i.e. $\eta^* > 0$). If k exceeds the knife-edge case, agent 2 wants to be negatively exposed to $\tilde{x}$, i.e. $\eta^* < 0$, as a hedge against future income shocks.

So what is an appropriate choice for the parameter k? Recall from equations (1.32) and (1.33) that k measures comovements between dividends and labor earnings, i.e. between the returns to labor and capital. It makes economic sense to conjecture that the ratio of dividends to labor earnings is constant in the long run, i.e. to assume that dividends and labor earnings are cointegrated. The long-run restriction that the factor shares of labor and capital are stationary is suggested by the form of most production functions used in macroeconomic theory. If labor and capital income are allowed to have independent trends (whether deterministic or stochastic), then the factor share of labor will approach zero asymptotically (if capital income grows faster than labor income) or the factor share of capital will approach zero (in the opposite case). This is contrary to what the data shows: although factor shares vary over time, they show no tendency to converge to zero or one. Indeed, Benzoni, Collin-Dufresne, and Goldstein (2007) provide empirical evidence that dividends and labor earnings are cointegrated. According to their empirical calibration, cointegration takes effect at an horizon of 5-20 years. That is: if dividends double in size over the next 5-20 years, then it can be expected that labor earnings will also approximately double in size over the same period. In our two-agent model, each period has a duration of 30 years, implying that dividends and labor earnings move together at a one-period horizon. From equations (1.32) and (1.33) it follows straightforwardly that the two-agent framework is consistent with the notion of cointegration by setting $k = 1$.

If $k = 1$, the total demand for the risky asset becomes negative, i.e. $\alpha^* < 0$. The negative demand for stocks by agent 2 (to hedge against future shocks in labor earnings) dominates the positive demand for stocks by agent 1. For the benchmark parameters, the total demand for the risky asset becomes negative if $k > 0.3$. A negative demand for the risk asset is feasible in this partial equilibrium framework, which takes the perspective of a small, open economy that is able to trade with foreign countries and is too small to affect the global price of risk. In a closed economy, however, a low risk appetite leads to an increase the equity premium, inducing agent 1 to take more risk. This result stands is sharp contrast to the finding in section 1.2.3 that risk sharing increases the demand for the risky asset, and hence reduces the equity premium. Although this dissertation focuses on risk sharing, comovements between stock and labor markets thus might have important implications for general equilibrium models that attempt to explain the equity premium puzzle.

The finding that cointegration leads to a negative demand for the risky asset can be interpreted in different ways. One interpretation is that risk sharing does not take place in practice, otherwise the equity premium in financial markets would be higher. Another interpretation is that the model overstates the effect of cointegration on the demand for stocks. There are several reasons for why this may be the case. First, the assumption that the variation in stock returns is fully driven by dividend shocks is inconsistent with the data. The variation in stock returns is subject to risk sources other than dividends shocks. Time-variation in discount rates is an important source of variation in stock-returns that potentially correlates less with human wealth returns.[13] Also irrationalities, such as mispricing and bubbles in asset prices, may be a source of variation in stock returns. The effect of cointegration on portfolio holdings may also be reduced by foreign stock holdings. The assumption that future labor earnings are cointegrated with dividends makes economic sense in the context of domestic stock holdings, but is less straightforward for foreign holdings. Emerging markets can have growth rates that deviate from those of developed countries over a sustained period of time.

For all the reasons above, the model may overstate the effect of cointegration on portfolio holdings. Therefore, let us introduce an additional source of variation in stock returns to the model. Let the return $\tilde{x}$ on period-1 stock holdings be

[13] Human may may be less affected by time-variation in discount rates due to its short duration. The duration of financial wealth is large: the stock price is based on the discounted value of an infinite flow of future dividends. The duration of human wealth, in contrast, is limited by the retirement date of an investor. The duration of the human wealth of a social planner is also limited, due to the inability to capitalize the labor earnings of future generations, see chapter 2.

specified as:

$$\tilde{x} = \mu + \tilde{z}_2 + \tilde{z}_3, \tag{1.37}$$

where $\tilde{z}_2$ and $\tilde{z}_3$ are random variables independent of each other and are distributed with mean zero and variance σ_2^2 and σ_3^2. The term $\tilde{z}_3$ represents the variation in stock returns that is due to shocks in future dividend levels. The term $\tilde{z}_2$ captures all the other sources of variation in stock returns. As explained above, these other sources of risk may include time-variation in discount rates, mispricing, asset bubbles or foreign stock holdings. The total variance of the stock return is denoted by σ^2, and it follows that $\sigma^2 = \sigma_2^2 + \sigma_3^2$. Hence, the extended model allows the volatility of stock returns to exceed the volatility of dividends, consistent with the data. Labor earnings, previously given by equation 1.32, are specified as:

$$\tilde{L}_2 = \bar{L}_2 \left(1 + k\tilde{z}_3\right), \tag{1.38}$$

If $k = 1$, labor earnings are cointegrated with dividends. Consistent with previous sections, the risk-sharing contract is assumed linear and conditioned upon the realization of the risky asset return $\tilde{x}$, i.e. $t(\tilde{x}) = t_0 + \eta\alpha\tilde{x}$. The optimization problem becomes:[14]

$$\max_{\alpha,\eta} \left\{ \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\left(\left(\eta\alpha + k\bar{L}_2\right)^2 \sigma_3^2 + (\eta\alpha)^2\,\sigma_2^2 \right) \right\}. \tag{1.39}$$

Agent 2's exposure to dividend shocks $\tilde{z}_3$ equals $\eta\alpha + k\bar{L}_2$, where $k\bar{L}_2$ represents the exposure via human wealth and where $\eta\alpha$ represents the exposure via risk sharing. In absence of other sources of variation in stock returns, i.e. $\sigma_2 = 0$, the problem reduces into equation (1.34). The optimal risk exposure α^* solves as:

$$\alpha^* = \frac{\mu}{\gamma\sigma^2}\left(\bar{L}_1 + \bar{L}_2\right) - \frac{\sigma_3^2}{\sigma^2}k\bar{L}_2. \tag{1.40}$$

Labor income risk causes the demand for the risky asset to reduce by an amount $(\sigma_3^2/\sigma^2)k\bar{L}_2$. Assuming $k = 1$, the effect of long-run labor income risk on the demand for the risky asset is determined by σ_3^2/σ^2: the fraction of the variation in stock returns that is due to dividend shocks. If dividend shocks are responsible for a larger fraction of stock return variation, portfolio returns become more strongly correlated with future labor earnings, and agent 2's demand for the risky asset decreases. The demand for the risky asset is positive, i.e. $\alpha^* > 0$, if less than 30% of the variation in stock returns is due to dividend shocks.

[14]The optimization problem in equation (1.39) is equivalent to that in equation (1.34), with k being replaced by $k\sigma_3^2/\sigma^2$.

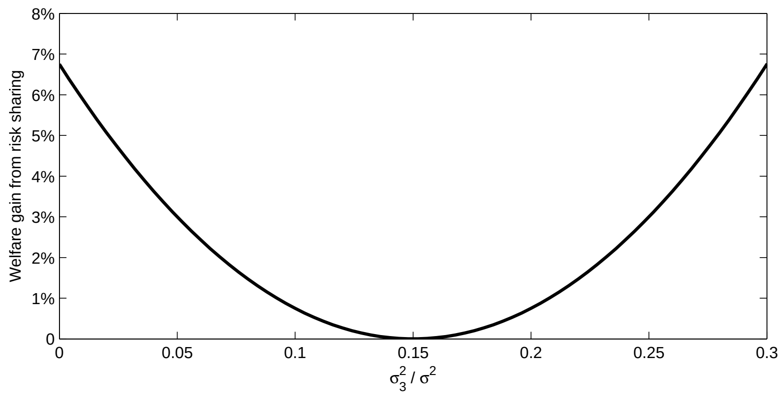


Figure 1.8: *The gains from risk sharing as a function of the fraction σ_3^2/σ^2 of stock return variation due to dividend shocks. Calculations are based upon the default parameters and $k = 1$.*

The knife-edge case in which autarky is Pareto-efficient is given by $k = \mu/(\gamma\sigma_3^2)$. Let us apply the numerical example parameters to this knife-edge situation. Assuming $k = 1$, the knife-edge case is given by $\sigma_3^2 = \mu/\gamma = 0.9/5 = 0.18$, implying that the gains from risk sharing are fully eroded if $\sigma_3^2/\sigma^2 = 0.18/1.1^2 = 15\%$ of the variation in stock returns is due to dividend shocks. This result is graphically illustrated in Figure 1.8. In absence of dividend shocks, i.e. $\sigma_3 = 0$, labor earnings and stock returns do not have a common risk factor and the gain from risk sharing equals 6.7% as in section 1.2.3. In the knife-edge case where σ_3^2/σ^2=15%, autarky is Pareto-efficient and the gains from risk sharing are fully eroded.

From this exercise it is inferred that comovements between stock and labor markets have a large impact on the gains from risk sharing, and that the effects are very sensitive to the decomposition of the variation in stock returns. Chapter 4 extends the analysis to a richer modeling environment.

1.3 Motivation of the remainder

Section 1.2.3 has shown that the gains from risk sharing are large if labor-market aspects are ignored. However, the analysis in sections 1.2.4 and 1.2.5 pointed out that these gains are reduced once labor-market distortions and the long-run dynamics of labor income are recognized. The highly stylized two-agent model, however, does not give reliable quantitative answers. The remaining chapters therefore extend the analysis to a richer modeling environment.

Chapters 2 and 3 are concerned with the welfare effects of labor-supply distortions, extending the analysis of section 1.2.4. Chapter 4 explores the effects of long-run labor income risk, extending the analysis of section 1.2.5. The difference between chapters 2 and 3 is as follows. Chapter 2 assumes simple but realistic pension fund policy rules. These policy rules are consistent with commonly observed policy practices around the world. It will be shown that, under these policy practices, the welfare costs from labor-market distortions can be very large. Therefore, chapter 3 examines to what extent labor-market distortions can be mitigated if the policy rules of the pension fund are fully optimized.

1.A Appendix

Proof of equation (1.5)

This section derives the Arrow-Pratt approximation in equation (1.5) along the lines of Gollier (2001). First, notice that the expected utility from consumption of agent 1 can be written as

$$\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\tilde{x}\right)^{1-\gamma}\right]=\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu+\alpha\hat{x}\right)^{1-\gamma}\right], \tag{1.41}$$

where

$$\hat{x}=\tilde{x}-\mu, \tag{1.42}$$

so that $\hat{x}$ is distributed with mean zero and variance σ^2. Let $\pi(\bar{L}_1,\gamma,\tilde{x}\alpha)$ denote the risk premium that is associated with the risk $\alpha\hat{x}$:

$$\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu+\alpha\hat{x}\right)^{1-\gamma}\right]\equiv\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu-\pi(\bar{L}_1,\gamma,\tilde{x}\alpha)\right)^{1-\gamma}. \tag{1.43}$$

The equation states that agent 1 is indifferent between bearing the risk $\hat{x}$ and paying the fixed risk premium. To simplify notation, let us define a function g as follows

$$g(\alpha)\equiv\pi(\bar{L}_1,\gamma,\tilde{x}\alpha), \tag{1.44}$$

so that

$$\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu+\alpha\hat{x}\right)^{1-\gamma}\right]=\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\mu-g(\alpha)\right)^{1-\gamma}. \tag{1.45}$$

Assuming the risk to be small, the effect of the expected return $\alpha\mu$ on the risk premium is negligible, equation (1.45) is approximated by:

$$\mathbf{E}\left[\frac{1}{1-\gamma}\left(\bar{L}_1+\alpha\hat{x}\right)^{1-\gamma}\right]\approx\frac{1}{1-\gamma}\left(\bar{L}_1-g(\alpha_i)\right)^{1-\gamma}. \tag{1.46}$$

The function g is approximated by a Taylor expansion around $\alpha=0$:

$$g(\alpha)\approx g(0)+\alpha g'(0)+\frac{1}{2}\alpha^2 g''(0). \tag{1.47}$$

It follows from equation (1.46) that

$$g(0)=0. \tag{1.48}$$

Differentiating equation (1.46) with respect to α_i yields

$$\mathbf{E}\left[\hat{x}\left(\bar{L}_1+\alpha\hat{x}\right)^{-\gamma}\right]=-g'(\alpha)\left(\bar{L}_1-g(\alpha)\right)^{-\gamma}, \tag{1.49}$$

from which it follows that

$$g'(0)=0, \tag{1.50}$$

since $\mathbf{E}[\hat{x}]=0$. Differentiating again with respect to α yields

$$-\mathbf{E}\left[\hat{x}_i^2\gamma\left(\bar{L}_1+\alpha\hat{x}\right)^{-\gamma-1}\right]=-g''(\alpha)\left(\bar{L}_1+g(\alpha)\right)^{-\gamma}+[g'(\alpha)]^2\gamma\left(\bar{L}_1+g(\alpha)\right)^{-\gamma-1}. \tag{1.51}$$

Evaluating this expression at $\alpha=0$ yields

$$g''(0)=\frac{\gamma}{\bar{L}_1}\sigma^2, \tag{1.52}$$

where it is used that $g(0)=0$ and $g'(0)=0$ and $\mathbf{E}\left[\hat{x}^2\right]=\sigma^2$. Substitution of equations (1.48), (1.50) and (1.52) into equation (1.47) yields

$$g(\alpha)\approx\frac{1}{2}\frac{\gamma}{\bar{L}_1}\sigma^2\alpha^2. \tag{1.47'}$$

Substitution of equations (1.46) and (1.47') into equation (1.41) yields equation (1.5).

Proof of equation (1.9)

Substitution of equation (1.6) into equation (1.4) yields the utility level U_1 of agent 1 in autarky under the optimal portfolio rule:

$$\begin{aligned} U_1^{aut} &= \mathbf{E}\left[\frac{(c_1)^{1-\gamma}}{1-\gamma}\right] = \mathbf{E}\left[\frac{(w_1+\alpha^{aut}\tilde{x})^{1-\gamma}}{1-\gamma}\right] \\ &\approx \frac{\left(w_1+\alpha^{aut}\mu-\frac{1}{2}\frac{\gamma}{w_1}(\alpha^{aut})^2\sigma^2\right)^{1-\gamma}}{1-\gamma} \\ &= \frac{\left(w_1+\frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}w_1\right)^{1-\gamma}}{1-\gamma} \end{aligned} \tag{1.53a}$$

Agent 2 is unable to share in period-1 risk in autarky and it follows from (1.1b) that:

$$U_2^{aut} = \frac{(w_2)^{1-\gamma}}{1-\gamma} \tag{1.53b}$$

In the presence of risk-sharing transfers between the two agents, utility levels are given by:

$$\begin{aligned} U_1(\alpha,t_0,\eta) &= \mathbf{E}\left[\frac{(c_1)^{1-\gamma}}{1-\gamma}\right] = \mathbf{E}\left[\frac{(w_1+\alpha\tilde{x}-t(\tilde{x}))^{1-\gamma}}{1-\gamma}\right] \\ &= \mathbf{E}\left[\frac{(w_1+(1-\eta)\alpha\tilde{x}-t_0)^{1-\gamma}}{1-\gamma}\right] \\ &\approx \frac{\left(w_1+(1-\eta)\alpha\mu-\frac{1}{2}\frac{\gamma}{w_1}(1-\eta)^2\alpha^2\sigma^2-t_0\right)^{1-\gamma}}{1-\gamma} \end{aligned} \tag{1.54a}$$

and

$$\begin{aligned} U_2(\alpha,t_0,\eta) &= \mathbf{E}\left[\frac{(c_2)^{1-\gamma}}{1-\gamma}\right] = \mathbf{E}\left[\frac{(w_2+t(\tilde{x}))^{1-\gamma}}{1-\gamma}\right] \\ &\approx \frac{\left(w_2+\eta\alpha\mu-\frac{1}{2}\frac{\gamma}{w_2}\eta^2\alpha^2\sigma^2+t_0\right)^{1-\gamma}}{1-\gamma} \end{aligned} \tag{1.54b}$$

Recall that the equivalent variation EQV_i for agent i (i being equal to 1 or 2) is defined as the amount of wealth that agent i should be given in the autarky case in order to obtain the level of utility U_i that will be achieved by participating in the risk sharing solution. Hence, it follows from combining equations (1.53) and (1.54) that:

$$EQV_1(\alpha, t_0, \eta) = (1-\eta)\alpha\mu - \frac{1}{2}\frac{\gamma}{w_1}(1-\eta)^2\alpha^2\sigma^2 - t_0 - \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}w_1, \tag{1.55a}$$

and

$$EQV_2(\alpha, t_0, \eta) = \eta\alpha\mu - \frac{1}{2}\frac{\gamma}{w_2}\eta^2\alpha^2\sigma^2 + t_0, \tag{1.55b}$$

Hence,

$$EQV_1 + EQV_2 = \alpha\mu - \frac{1}{2}\frac{\gamma}{\bar{L}_1}(1-\eta)^2\alpha^2\sigma^2 - \frac{1}{2}\frac{\gamma}{\bar{L}_2}\eta^2\alpha^2\sigma^2. \tag{1.56}$$

Proof of equation (1.25)

Equation (1.25) is derived along the same lines as equation (1.5) was derived, using the Arrow-Pratt approximation. The expected utility from consumption of agent 2 can be written as

$$\begin{aligned} & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2h_2 + \alpha\tilde{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \\ = \; & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2h_2 + \alpha\mu + \alpha\hat{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \end{aligned} \tag{1.57}$$

where h_2 represents the labor-supply choice of agent 2 and is a stochastic variable (and will be defined later in equation (1.63)) and where

$$\hat{x} = \tilde{x} - \mu, \tag{1.58}$$

so that $\hat{x}$ is a pure risk distributed with mean zero and variance σ^2. Let $\pi(w_2h_2, \gamma, \tilde{x}\alpha_, \epsilon)$ denote the risk premium that is associated with the risk $\alpha\hat{x}$:

$$\begin{aligned} & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2h_2 + \alpha\mu + \alpha\hat{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \\ \equiv \; & \frac{1}{1-\gamma}\left(w_2h_2^* + \alpha\mu - \pi(w_2h_2, \gamma, \tilde{x}, \alpha, \epsilon)\right)^{1-\gamma} \end{aligned} \tag{1.59}$$

Thus, $\pi(w_2h_2, \gamma, \tilde{x}, \alpha, \epsilon)$ denotes the risk premium that makes agent 2 indifferent between bearing the pure risk $\hat{x}$ on the one hand and paying the fixed risk premium and facing no labor-supply distortions on the other hand. To simplify notation, let us define a function g as follows

$$g(\alpha) \equiv \pi(w_2 h_2, \gamma, \tilde{x}, \alpha, \epsilon), \tag{1.60}$$

so that

$$\begin{aligned} & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2 h_2 + \alpha\mu + \alpha\hat{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \\ \equiv \;& \frac{1}{1-\gamma}\left(w_2 h_2^* + \alpha\mu - g(\alpha)\right)^{1-\gamma} \end{aligned} \tag{1.61}$$

Assuming the risk to be small, the effect of the expected return $\alpha\mu$ on the risk premium is negligible, so that equation (1.61) is approximated by:

$$\begin{aligned} & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2 h_2 + \alpha\hat{x} - \frac{\epsilon}{\epsilon+1}(h_2)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}(h_2^*)^{\frac{\epsilon+1}{\epsilon}}\right)^{1-\gamma}\right] \\ \approx \;& \frac{1}{1-\gamma}\left(w_2 h_2^* - g(\alpha)\right)^{1-\gamma} \end{aligned} \tag{1.62}$$

The stochastic variables for respectively the labor-supply choice h_2 and the tax or subsidy on labor supply $\tilde{\tau}$ are functions of each other and do not attain an explicit solution. To arrive at an explicit expression, let the labor-supply choice be approximated by

$$h_2 = h_2^*(1+\tilde{\tau})^\epsilon = h_2^*\left(1 + \frac{\alpha\tilde{x}}{w_2 h_2}\right)^\epsilon \approx h_2^*\left(1 + \frac{\alpha\hat{x}}{w_2 h_2^*}\right)^\epsilon, \tag{1.63}$$

Substitution of equation (1.63) into equation (1.62) yields

$$\begin{aligned} & \mathbf{E}\left[\frac{1}{1-\gamma}\left(w_2 h_2^*\left(1 + \frac{\alpha\hat{x}}{w_2 h_2^*}\right)^\epsilon + \alpha\hat{x} - \frac{\epsilon}{\epsilon+1}\left(h_2^*\right)^{\frac{\epsilon+1}{\epsilon}}\left(\left(1 + \frac{\alpha\hat{x}}{w_2 h_2^*}\right)^{\epsilon+1} - 1\right)\right)^{1-\gamma}\right] \\ \equiv \;& \frac{1}{1-\gamma}\left(w_2 h_2^* - g(\alpha)\right)^{1-\gamma} \end{aligned} \tag{1.64}$$

The function g is approximated by a Taylor expansion around $\alpha = 0$:

$$g(\alpha) \approx g(0) + \alpha g'(0) + \frac{1}{2}\alpha^2 g''(0). \tag{1.65}$$

It follows from equation (1.64) that

$$g(0) = 0. \tag{1.66}$$

Differentiating equation (1.64) with respect to α and evaluating the resulting expression at $\alpha = 0$ yields

$$g'(0) = 0. \tag{1.67}$$

Differentiating equation (1.64) twice with respect to α and evaluating the resulting expression at $\alpha = 0$ yields

$$g''(0) = \frac{\gamma + \epsilon}{w_2 h_2^*}\sigma^2, \tag{1.68}$$

where it is used that $g(0) = 0$ and $g'(0) = 0$ and $\mathbf{E}\left[\hat{x}^2\right] = \sigma^2$. Substitution of equations (1.66), (1.67) and (1.68) into equation (1.65) yields

$$g(\alpha) \approx \frac{\gamma + \epsilon}{2 w_2 h_2^*}\alpha^2\sigma^2. \tag{1.65'}$$

Substitution of equations (1.61) and (1.65') into equation (1.57) yields equation (1.25).

Chapter 2

Risk Sharing under Endogenous Labor Supply

This chapter examines how the gains from risk sharing are affected by the presence of labor-market distortions. The results in this chapter show that the costs from labor-market distortions are of first-order importance in an assessment of risk-sharing. The model extends the stylized two-agent framework in section 1.2.4. The analysis abstracts from labor income risk, which is the topic of chapter 4.

In this chapter, the term "endogenous" labor supply does *not* refer to the use of labor supply as a buffer against income shocks, as studied in the Bodie, Merton, and Samuelson (1992). In Bodie, Merton, and Samuelson (1992), a negative (positive) wealth shock increases (decreases) the marginal utility from working and hence agents increase (reduce) labor supply, implying that *income* effects in labor supply work as a buffer against wealth shocks. Instead, this chapter examines *substitution* effects in labor supply which appear in the context of a pension fund in which financial shocks are levied (in part) in the form of implicit taxes and subsidies on labor earnings.

2.1 Introduction

Intergenerational risk sharing makes it possible for future generations to share in current risk. Current risk can be shifted into the future via transfers that redistribute wealth between generations. If properly designed, risk-sharing contracts improve the welfare of all generations from an ex-ante perspective. From an ex-post perspective, however, future generations are worse off in the case where they have to transfer a part of their wealth to currently-living generations. Hence, future generations must be committed to the contract in order to make intergenerational risk-sharing feasible.

Pension funds are able to commit future generations by imposing participation in the fund to be mandatory for all workers. Future generations can be required to join a pension scheme with a funding deficit, which forces them to share in the financial losses of previous generations. A pension fund is able to recoup previous losses upon current generations by charging contribution rates that are high in relation to the value of pension entitlements accrued in return. The ability of a pension fund to induce a wedge between the contribution rate and the accrual rate makes it possible to levy implicit taxes or subsidies on labor earnings. If the contribution rate exceeds the accrual rate, the pension fund levies an implicit tax on the labor earnings of its workers. A net subsidy is provided in the opposite case. This "power of taxation" provides pension funds with the ability to collateralize future labor earnings, thereby allowing it to make commitments on behalf of unborn generations.

This chapter recognizes that it can be difficult to commit future generations to a risk-sharing contract, even if participation is mandatory. A wealth transfer from future generations to currently-living ones results in a claim on future labor income, and hence discourages labor supply. High contribution rates discourage work, and hence provide workers with an incentive to reduce their number of working hours, to retire early, or to move into the grey or black economy. Indeed, there is a vast literature that finds that the labor-supply choices of workers to be quite responsive to the financial incentives in pension schemes, see e.g. Stock and Wise (1990), Samwick (1998) and Gruber and Wise (1999). A pension fund thus faces a trade-off between risk sharing and labor-supply distortions. In the context of an occupational pension fund, risk sharing not only leads to distortions in labor *supply*, but also to distortions in labor *mobility*. Workers are able to get around high contribution rates by switching to another employer (in the case of a corporate scheme) or switching to another industry (in the case of an industry-wide scheme) with an actuarially fair pension scheme.

This chapter evaluates the trade-off between risk sharing and labor-market distortions. The central feature of the model is that the financial gains and losses from risky investments by the pension fund result in implicit taxes and subsidies on the labor earnings of participants. A drop in the value of pension-fund assets can lead to a rise in the contribution rate, a cut in benefit levels, or a combination of the two. All other things equal, a rise in the contribution rate reduces the effective wage rate of participants, and hence discourages labor supply. Similarly, a benefit cut, if permanent in nature, leads workers to anticipate lower benefit levels in the future and thus reduces the value to pension entitlements accrued at present. By letting the contribution rate deviate from the value of pension entitlements accrued in return, the pension fund induces a wage-differential: it levies a net tax or provides a net subsidy on the labor earnings of its working participants.[1] The labor-supply choices of participants are assumed responsive to these indirect taxes and subsidies.

Without exception, previous studies on risk sharing in pre-funded pension schemes abstract from labor-market distortions. Risk-sharing contracts that solely rely on lump-sum transfers, however, are unrealistic in the context of intergenerational risk-sharing. This is due to the fact that the individuals within a generation are heterogenous in terms of their earnings-capacity. That is: each generation consists of both high-ability individuals with a high earnings capacity as well as low-ability individuals with a low earnings capacity. Individuals with a high earnings-capacity are typically assumed to have a higher risk-bearing capacity than individuals with a low earnings capacity. For example, if all individuals have the same degree of relative risk aversion, then the optimal exposure to economic shocks is typically proportional to wealth. The optimal risk-sharing solution thus features the property that individuals within cohorts are unequally affected by economic shocks in absolute terms: high-ability individuals absorb a larger part of a shock (in absolute terms) than low-ability individuals. Hence, the absolute size of risk sharing transfers varies among individuals within a cohort in an optimal risk-sharing solution.

Ideally, a social planner would like to apply lump-sum transfers that do not distort the labor market when transferring wealth between generations. In an ideal

[1]Not *all* financial shocks manifest themselves in the form of taxes and subsidies. If the pension fund recovers from a financial loss via an unanticipated cut in benefit levels, then retirees experience this as a lump-sum transfer.

system of taxation, lump-sum taxes and subsidies can be related to the earnings capacity of economic agents. Tinbergens proposal of a “tax on talent” follows this argument, see Tinbergen (1970) and Tinbergen (1975). However, the consensus in the literature is that the government does not know enough about individuals to determine their lump sums. The literature on optimal taxation that has emerged in the wake of the seminal contribution of Mirrlees (1971) indeed starts out from the information problem.[2]

Heterogeneity in earnings-capacities is not explicitly modeled in order to keep the analysis as simple and transparent as possible. Although not explicitly included in the model, the reader should bear in mind that unobserved heterogeneity in labor earnings is the essential motivation for labor-market distortions, as explained in section 1.1.3. The gains from risk sharing are evaluated along the lines of Gollier (2008), in which a benevolent pension fund maximizes the ex-ante welfare of all generations. The pension fund dynamically determines the optimal policy with respect to contribution rates, benefit payments and portfolio allocations. The risk-sharing solution is compared to the “autarky” solution in which the economic agents save and invest in an individual retirement account. The pension fund is stand-alone, in the sense that there is no risk-absorbing sponsor in the form of a corporation.

The four main findings of this chapter are as follows. First, I show that the costs from labor-market distortions are of first-order importance in a risk-sharing assessment. For the benchmark parameters, the gains from risk sharing are fully eroded by distortions if the compensated elasticity of labor supply exceeds the value of 1.1. In this situation, the costs from distortions dominate the gains from risk sharing, implying that workers are better off in a system with individual retirement accounts.[3]

[2]Arguably, moral hazard causes lump-sum transfers to be infeasible *even* if the earnings capacity of participants is perfectly observed. Economic agents typically enjoy limited-liability, implying that the size of wealth transfers is restricted by the effort level chosen by participants. Hence, the promise to transfer wealth from future generations to current ones becomes difficult to enforce, because a large claim on labor income provides workers with an incentive to default.

[3]The model features the property that a Pareto-efficient risk-sharing solution does not exist if the elasticity of labor supply is high. This modeling feature is due to the simple benefit rule that is used in this chapter. Simple benefit rules are consistent with the policy practice of pension funds around the world. Chapter 3 derives the optimal risk-sharing contract in absence of any restrictions imposed on the policy rules of the pension fund.

The second finding is that a pension fund faces a trade-off between consumption smoothing on the one hand and minimizing labor-market distortions on the other hand. Consumption smoothing is a familiar finding in the literature, and teaches that wealth shocks should be smoothed over as many periods or generations as possible. [4] I show consumption smoothing is not optimal anymore once the costs from labor-market distortions are recognized. Under the principle of consumption smoothing, every economic shock results in a permanent adjustment of consumption. As explained before, these adjustments in consumption levels have to be implemented via distortionary taxes. The permanent nature of adjustments in consumption levels implies that also adjustments in the tax rate are permanent under the principle of consumption smoothing. As a result, the distortions induced by current shocks add to already existing distortions from *all* previous shocks that have occurred in the past. It is well-known from the intuition of the Harberger triangle that the marginal costs from current distortions are higher if they add to already-existing distortions. Hence, the welfare costs from distortions become very large in the long run if every distortion in the labor market is permanent in nature.

As a result, consumption smoothing is not feasible in the presence of labor-market distortions. Instead, it is optimal for financial shocks to be levied primarily upon currently-living generations, instead being smoothed over as many generations as possible. By recouping shocks in the short-run, a pension fund restores its capacity to absorb new shocks in the future. Hence, it is optimal for a pension fund to recover from shocks in a relatively short time-period, consistent withsolvency regimes for pension fund that are imposed by regulators in some countries. The Dutch regulator, for example, requires pension funds with a funding deficit to recover within 3 years and to have restored their financial buffer for risk-taking within 15 years.

At first sight, this result appears to be inconsistent with the literature on *tax-smoothing*. This literature finds that governments should set their debt-policies in such a way that distortionary taxes are smoothed over time, see e.g. Barro (1979).

[4] Consumption smoothing has been found to be optimal in a model with a finitely-lived investor (Samuelson (1969) and Merton (1969)), in a model with an infinitely-lived consumer (Hall (1987)), in a risk-sharing solution in which non-overlapping generations trade with each other in a fictitious financial market (Ball and Mankiw (2007)), and in a risk-sharing solution in which a social planner reallocates risk across non-overlapping generations (Gollier (2008)).

However, I find that the solution derived in this chapter, in which current shocks are primarily levied upon currently-living generations, is in fact consistent with the basic principle of tax-smoothing. If all shocks are perfectly smoothed over future consumption levels via a permanent adjustment in taxes, then each economic shock induces a distortion that is permanent in nature, implying that later-born generations face more shocks and thus face higher distortions in comparison to earlier-born ones. Hence, in order to distribute (or smooth) the burden of distortions equally over generations, current generations absorb a disproportionately large fraction of current shocks.

As the third finding of this chapter, I show that labor-supply flexibility (i.e. the ability of workers to vary labor supply ex-post) can reduce the appetite for risk taking. This result stands in striking contrast with Bodie, Merton, and Samuelson (1992), who find that labor-supply flexibility increases the risk bearing capacity for the case of an individual investor.[5] For an individual investor, a negative (positive) wealth shock increases (decreases) the marginal utility from working and hence agents increase (reduce) labor supply. These income effects in labor supply works as a buffer against wealth shocks, and lead to counter-cyclical labor-supply behavior. Thereby, labor supply flexibility enables an individual investors to take greater advantage of the risk premium in financial markets. In contrast, this paper evaluates asset management at a pension fund rather than the portfolio holdings of an individual investor. In a pension fund, financial shocks are levied, in part, in the form of implicit taxes and subsidies on labor earnings. These distortionary transfers are necessary because of the unobserved heterogeneity in earnings capacities within cohorts, as explained in section 1.1.3. The financial incentives of pension contracts not only induce income effects in labor supply, as in the analysis of Bodie, Merton, and Samuelson (1992), but also substitution effects. Substitution effects in labor supply work in the opposite direction as income effects, and induce pro-cyclical labor-supply behavior. While an increase in income-elasticity in labor supply increases the risk-bearing capacity of the pension fund, an increase in substitution-elasticity in labor supply reduces it. Empirical studies (see e.g. Blundell and MaCurdy (1999)) find that substitution effects in labor supply dominate income effects. If this is the case, then the financial incentives in pension

[5] The analysis of Bodie, Merton, and Samuelson (1992) has been further developed by, among others, Farhi and Panageas (2007), Choi, Shim, and Shin (2008) and Gomes, Kotlikoff, and Viceira (2008).

plans lead to pro-cyclical labor-supply behavior, which reduces the risk-bearing capacity of a pension fund.

As its fourth contribution, this chapter provides a more accurate assessment of risk sharing in comparison to earlier studies. In particular, the assumption of "defined contributions" made in Gollier (2008) is relaxed. This chapter allows the savings rate of workers to vary across time in response to income shocks. Flexible contribution rates dramatically increase the risk-bearing capacity of workers, and enables the pension fund to take more advantage from risk sharing. I find that the gains from risk sharing roughly become twice as large once the assumption of defined-contributions is relaxed.

Many papers have studied intergenerational risk-sharing in the context of a pay-as-you-go financed pension scheme, see e.g. Bohn (1998), Krueger and Kubler (2002) and Gottardi and Kubler (2008).[6] Only few papers have studied the risk-sharing aspects of pre-funded pension schemes, see. e.g. Teulings and De Vries (2006), Cui, De Jong, and Ponds (2011), Ball and Mankiw (2007) and Gollier (2008). These papers, however, ignore the effects of risk sharing on labor and capital markets. Beetsma and Bovenberg (2009) examine the implications of risk sharing for capital markets but ignore labor-market effects.[7]

It is possible to view the modeling framework of this chapter in a "broader" perspective. In the strict interpretation of the model, the commitment problem of the pension fund manifests itself in the form of labor supply responses, which make it difficult to tie workers to the insurance pool of the pension fund. In this strict interpretation of the model, workers are said to be"voting with their feet", thereby making it difficult for a pension fund to commit workers to risk sharing.

[6] A number of papers have studied a transition from a pay-as-you-go financed pension scheme towards a funded scheme, see e.g. Krueger and Kubler (2006), Nishiyama and Smetters (2007), Fuster, Imrohoroglu, and Imrohoroglu (2007), Fehr and Habermann (2008). All these studies, however, take the perspective of a transition towards a individual retirement accounts, and thus ignore the possibilities for risk sharing in a pre-funded pension scheme.

[7] **?** also examine the labor-supply distortions from risk sharing in the context of a funded pension scheme. Their analysis is restricted to the case of a stylized two-agent model and provides analytical results only for the case of Cobb-Douglas preferences over consumption and leisure. Their quantitative results are consistent with the welfare losses of the two-agent model in chapter 1: 5-20% of the social surplus from risk sharing is eroded by distortions. I show that quantitative results are substantially different in a framework that is able to incorporate realistic pension fund policy-rules.

In a “broader” interpretation, however, the model also accounts for other ways in which the commitment problem can manifest itself. For example, participants may not just “vote by using their feet”, but they can also “vote by using their voice”, in which case they threaten to terminate the pension fund if this is in their interest.[8] In this situation, the commitment problem manifests itself in the form of discontinuity risk for the pension fund.

The remainder of this chapter is as follows. Section 2.2 introduces the model. Section 2.3 treats the autarky problem in which individuals save and invests on an individual retirement account. Section 2.4 discusses the risk-sharing solution. Finally, section 2.5 concludes.

2.2 The model

I extend the modeling framework of Gollier (2008) to the case of elastic labor-supply. In contrast to Gollier (2008), I do not assume defined-contributions. The assumption of constant consumption during the working period is problematic because this assumption introduces an additional distortion in labor-supply choices. Without the defined-contributions assumption, an economic shock results in adjustment of all future consumption levels: those during the working period as well as those during retirement. In a system with defined contributions, in contrast, an economic shock leads to an adjustment in retirement consumption, while consumption during the working period remains unaffected. Imperfect consumption smoothing provides workers with an incentive to adjust their labor supply in order to accomplish a better smoothing of consumption over the life-cycle. Hence, the assumption of defined-contributions introduces a distortion in labor-supply choices.

[8] There are two scenarios in which participants financially benefit from termination of the pension scheme. Current generations have an incentive to abolish the risk-sharing contract if they are required to transfer wealth to future generations. In this situation, current generations consume the buffer of the pension fund themselves instead of passing it to future cohorts. Second, future generations have an incentive to abolish the risk sharing contract if they are required to transfer a part of their wealth to current generations. This situation corresponds to the case in which future generations refuse to join a pension fund with a funding deficit. This scenario is discussed in Gollier (2008), who notes that risk sharing “*becomes hardly sustainable in the event of a succession of negative shocks*”.

The economy

The model for the economy is partial equilibrium in the sense that wages and asset returns are exogenous. The partial equilibrium framework is consistent with a small open economy that is too small to affect world factor prices, and greatly reduces the complexity of the model. I adopt the standard Black-Scholes-Merton setting, which is briefly summarized below. The financial market offers two investment opportunities: a riskless asset and a stock. The return on both assets is assumed exogenous. The riskless asset yields an instantaneous real return r. The excess-return X_t on the investment strategy that reinvests all proceeds (dividends and capital gains) in the stock is given by:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dZ_t, \tag{2.1}$$

where Z_t is a standard Wiener process and where X_t refers to the return on stocks holdings in excess of the riskfree rate r. Applying Ito's lemma, it follows that the excess return on stocks is lognormally distributed:

$$X_v = X_t e^{\left(\mu - r - \frac{1}{2}\sigma^2\right)(v-t) + \sigma(Z_v - Z_t)}, \tag{2.2}$$

for all $v > t$, and that μ represents the expected excess return on stocks:

$$\mathbf{E}\left[X_v\right] = X_t e^{(\mu - r)(v-t)}, \tag{2.3}$$

for $v > t$. Let the risk premium $\lambda \equiv \mu/\sigma$ be defined as the expected return on stocks over the riskfree rate per standard deviation of the excess return (also referred to as the Sharpe-ratio). The financial market offers a trade-off between risk and return: investments in the risky asset introduce uncertainty while at the same time increasing the expected return on investments. Finally, let us specify the stochastic discount factor M_t of the economy. The market value at time t of a stochastic payoff P_v at time $v \geq t$ is given by: $\mathbf{E}_t\left[(M_v/M_t)P_v\right]$, where the stochastic discount factor obeys the following stochastic differential equation:

$$\frac{dM_t}{M_t} = -rdt - \lambda dZ_t. \tag{2.4}$$

Overlapping generations framework

Consider an overlapping generations model in which each generation works for a period of 40 years and is subsequently retired for a period of 20 years. During each period, there are $40 + 20 = 60$ overlapping generations alive. We are thus considering a 60-period overlapping generations (OLG) model. At each discrete point in time $t \in \mathbb{N}$, the oldest cohort passes away and a new generation enters the workforce. Furthermore, let "cohort s" refer to the cohort that enters the labor market at time $s \in \mathbb{N}$. All cohorts are assumed to be equal in size, and the size of cohorts is normalized to unity. Individuals supply labor during the working period while being retired thereafter. The annual real wage rate per unit of labor supply, denoted by w, is assumed exogenous and constant and is constant over time and the same for each generation. Chapter 4 relaxes the assumption of a constant wage rate.

Individual preferences

The analysis abstracts *intra*-generational differences, implying that consumption and labor supply varies across time and cohorts, but not within cohorts. Let $C_{s,t}$ and $h_{s,t}$ denote consumption and labor supply at time t of an individual in cohort s. Preferences are given by time-additive utility from consumption and labor supply:

$$U_s = \mathbf{E}_0 \left[\int_s^{s+40} e^{-\beta(t-s)} u\left(C_{s,t}, h_{s,t}\right) dt + \int_{s+40}^{s+60} e^{-\beta(t-s)} u(C_{s,t}) dt \right], \tag{2.5}$$

for all cohorts s, where β represents the subjective time-discount factor of the individual and where the felicity function u is specified as:

$$\begin{cases} u(C_{s,t}, h_{s,t}) & = \frac{1}{1-\gamma} \left(C_{s,t} - \frac{\epsilon}{\epsilon+1} \left(h_{s,t}\right)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1} \left(h^*\right)^{\frac{\epsilon+1}{\epsilon}} \right)^{1-\gamma} \\ u(C_{s,t}) & = \frac{1}{1-\gamma} C_{s,t}^{1-\gamma} \end{cases} \tag{2.6}$$

where h^* represents the optimal labor-supply level in absence of distortions to the marginal wage rate:

$$h^* = w^{\epsilon}. \tag{2.7}$$

The felicity function in equation (2.6) was introduced in section 1.2.4, where it has been explained that there are no income-effects in labor supply and that ϵ denotes the compensated wage-elasticity of labor supply. If distortions in labor supply are not too large, the parameter γ approximately equals the relative risk aversion with respect to consumption $C_{s,t}$, regardless of the parameter choice for

ϵ. In the special case where labor supply is inelastic or undistorted (i.e. if $h_{s,t} = h^*$ for all s and all t), the specification simplifies into standard time-additive CRRA utility over consumption $C_{s,t}$.

The constant term $\frac{\epsilon}{\epsilon+1}\left(h^*\right)^{\frac{\epsilon+1}{\epsilon}}$ in equation (2.6) prevents a sudden drop in consumption patterns around retirement. If this constant term would not be added, then individuals would reduce their consumption level at the retirement date due to the increase in the leisure level during retirement. To simplify analytical expressions, my analysis abstracts from such a drop in consumption around retirement.

The absence of income effects in labor supply is assumed to preserve the analytically tractability of solution. Income effects in labor supply have been studied in Bodie, Merton, and Samuelson (1992) and work in the opposite direction as substitution effects. Hence, the parameter of labor supply elasticity ϵ should be interpreted as the *compensated* wage-elasticity of labor supply, i.e. the elasticity of labor supply corrected for income effects. The analysis focusses on substitution effects in labor supply, because these are essential in an assessment of risk sharing: a pension fund induces substitution effects in labor supply, whereas there are no substitution effects in the case where an individual invests on an individual account. Income effects in labor supply, in contrast, are more or less the same for a pension fund participant and an individual investor. For example, if the risk exposure of a pension fund participant is the same as the risk exposure of an individual investor, then the income effect in labor supply induced by an economic shock is the same. Hence, in this case the difference between the labor-supply choice of an individual investor and a pension fund participant is due to substitution effects induced by the pension fund.

Notice that adjustments in labor supply is restricted to decision of the number of hours worked. The retirement age is assumed to be exogenous in the model. This assumption is made to preserve the analytically tractability of the modeling solution. In the context of pension funds, the retirement decision is likely to be the most important channel through which workers adjust their labor supply decisions. Indeed, there is a vast literature that finds that the labor-supply choices of workers to be quite responsive to the financial incentives in pension schemes, see e.g. Stock and Wise (1990), Samwick (1998) and Gruber and Wise (1999). The reader should therefore bear in mind that the distortions induced by labor supply choices need to be interpreted in a broad way, also measuring the distortions retirement behavior.

Default parameters

There is a vast empirical literature that estimates the wage-elasticity of labor-supply ϵ, see e.g. Blundell and MaCurdy (1999), Alesina, Gleaser, and Sacerdote (2005) and Meghir and Phillips (2010). There is consensus in the literature that labor supply at the intensive margin (i.e. choices about hours of work or weeks of work) is not particularly responsive to financial incentives for male workers, but a little more responsive for married woman and lone mothers. On the other hand, labor supply choices at the extensive margin (i.e. the decision to participate in the labor-force participation at all) are quite sensitive to financial incentives. Also in the context of a pension fund, labor supply choices at the extensive margin play a dominant role. In particular, there is a substantial literature that finds the retirement decisions of workers to be quite responsive to financial incentives in pension schemes, see e.g. Stock and Wise (1990), Samwick (1998) and Gruber and Wise (1999).

The choice for ϵ depends crucially on the context of the problem at hand. In the context of a nation-wide pension fund, the policy of the pension fund can only be evaded by going abroad or by moving into the gray or black economy. Hence, the labor-supply responses induced by a nation-wide pension fund will be of the same order-of-magnitude as those induced by government tax policies. In this context, it is reasonable to say that labor supply responses are relatively modest. In this context, parameter choices for ϵ in the range of 0.2 to 0.5 can be considered reasonable. However, the introduction of this chapter has explained that labor-supply responses can be much more responsive in the context of employer-based or industry-based pension funds, which can be relatively easily evaded by switching jobs to a different company or a different firm. In such a context, assuming a high value for ϵ, for example in the range 0.5 to 1.5, can be realistic. In particular, it can be difficult to tie workers to the insurance pool of a pension fund if the human capital of workers is not very specific to the firm or the industry in which they are employed. In such a context, labor-supply choices of workers can be very responsive, making it difficult for a pension fund to extract quasi-rents.

Since the parameter choice for ϵ depends crucially on the context of the problem at hand, no default parameter for ϵ is specified in this dissertation. Instead, results are reported for a broad range of parameter choices. In some illustrative examples in this chapter, ϵ is chosen to be 1.0. In this situation, the results refer to a

r	0.02	riskfree rate
σ	0.2	volatility of stock returns
μ	0.03	expected excess return on stocks
β	0.02	subjective discount factor
γ	5	coefficient of relative risk aversion

Table 2.1: *Default model parameter values of chapters 2 and 3.*

situation in which the pension fund finds it difficult to tie workers to the insurance pool of fund, for example in the context of an employer-based or industry-based pension fund.

The default parameter choices for the other model variables are contained in Table 2.1.

2.3 Autarky

As a benchmark, let us first consider the autarky-situation where individuals save and invest on an individual retirement account. Since preferences, investment opportunities and real wages do not vary across time, the optimization problem in autarky is the same for all cohorts and does not to be solved for each cohort separately.

2.3.1 Optimization problem

During the working period, the investor makes intertemporal decisions with respect to consumption, labor-supply and investments. During the retirement period, only the consumption and investment choices remain. Since labor-supply choices are undistorted in autarky, preferences collapse into standard time-additive CRRA utility over consumption. The optimization problem reduces into the well-known problem of Samuelson (1969) and Merton (1969). For completeness, the solution to this problem is briefly discussed below.

An autarky-investor in cohort s maximizes preferences:

$$U_s = \max_{\alpha,C} \left\{ \mathbf{E}\left[\int_s^{s+60} e^{\beta(t-s)} \frac{1}{1-\gamma} \left(C_{s,t}\right)^{1-\gamma} dt \right] \right\}, \tag{2.8}$$

subject to the budget constraints:

$$\begin{aligned} dF_{s,t} &= rF_{s,t}dt + \alpha_{s,t}dX_t/X_t + wh^*dt - C_{s,t}dt && \text{for } t-s<40, \quad (2.9a)\\ dF_{s,t} &= rF_{s,t}dt + \alpha_{s,t}dX_t/X_t - C_{s,t}dt && \text{for } t-s\geq 40, \quad (2.9b)\\ F_{s,s} &= 0, && (2.9c)\\ F_{s,s+60} &= 0, && (2.9d) \end{aligned}$$

where $F_{s,t}$ and $\alpha_{s,t}$ denote respectively financial wealth and the amount invested in stocks of an individual of cohort s at time t. Equation (2.9d) specifies that there is no bequest motive.

2.3.2 Solution

The optimization problem of section 2.3.1 adopts an analytical solution, which is derived in Appendix 2.A. In the optimal solution, consumption choices are characterized by consumption smoothing. That is: wealth shocks are levied proportionally equally over all remaining consumption levels in the life-cycle:

$$\frac{\partial C_{s,v}}{C_{s,v}} = \frac{\partial W_{s,t}}{W_{s,t}}, \tag{2.10}$$

for all $v > t$, where $W_{s,t}$ denotes the total wealth of an investor in cohort s at time t and is defined as the sum of financial wealth and human wealth, i.e. $W_{s,t} = F_{s,t} + H_{s,t}$, with human wealth defined as the discounted value of future labor earnings: $H_{s,t} = \int_t^{s+40} e^{-r(v-t)} wh^* dv$ if $t - s < 40$ and zero otherwise. According to equation (2.10), a change in total wealth by $y\%$ percent results in a decline in all remaining consumption levels by $y\%$ percent. Shocks are thus smoothed over as many periods as possible, instead of being recouped in a few periods. This argument has been proposed by Bovenberg, Nijman, Teulings, and Koijen (2007) to justify the optimality of hybrid pension systems that adjust both contributions and benefits in response to income and wealth shocks. Pension plans that keep contributions fixed (a defined-contribution system) or plans that fix the benefits (a defined-benefit system) are not optimal in their view.

Portfolio allocations in the optimal solution are characterized by the property that stock investments are a constant fraction $\mu/(\gamma\sigma^2) = \lambda/(\gamma\sigma)$ of total wealth:

$$\alpha_{s,t} = \frac{\lambda}{\gamma\sigma} W_{s,t}, \tag{2.11}$$

for all s and all t. Thereby, the portfolio choice in equation (2.11) is consistent with expression (1.6) derived in the two-agent setting of chapter 1. During the beginning of the life-cycle, the wealth of the autarky investor consist primarily of human wealth. As the investor approaches retirement, the human wealth is gradually depleted and financial assets become the dominant wealth-component.

In the optimal solution, all generations are proportionally equally affected by an economic shock. Indeed, it follows from equations (2.10) and (2.11) that:

$$\frac{\partial W_{s,t}/W_{s,t}}{\partial Z_t} = \frac{\lambda}{\gamma}, \tag{2.12}$$

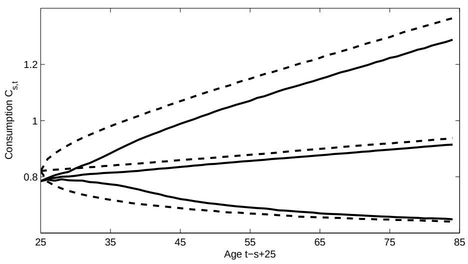
for all s and for all t.

In a 'defined contribution'-setting as in Gollier (2008), the expression for the optimal portfolio choice is the same, except for $H_{s,t}$ being replaced by the discounted value of future *savings* instead of future labor earnings. Given that savings are only a small fraction of earnings, the demand for stocks is substantially lower in a setting with defined-contributions. For example, the demand for stocks of a young investor without financial wealth reduces by 80% under defined-contributions assumption with a contribution rate of 20%.

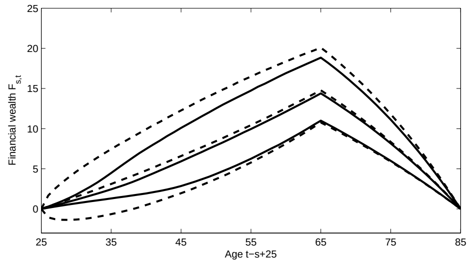
The dashed lines in Figure 2.1 show optimal solution for the benchmark parameters. The consumption smoothing property of the optimal solution causes consumption to follow a random-walk, because each consumption shock is smoothed over all future time-periods. Financial wealth is roughly increasing during the working period and decreasing during retirement. The demand for stocks of young investors exceeds the demand of their older counterparts, which is due to the large human wealth of the young. The dashed line in Figure 2.2 shows the share of current financial wealth invested in stocks, and illustrates that it is usually optimal to reduce the share of current financial wealth invested in equity (i.e. $\alpha_{s,t}/F_{s,t}$) over the working life. This argument has been proposed by Bodie, Merton, and Samuelson (1992) to justify the standard recommendation to reduce portfolio risk as one approaches the retirement age.

Welfare levels are expressed in terms of the percentage change in the certainty-equivalent consumption level C_s^{ce}, defined as:

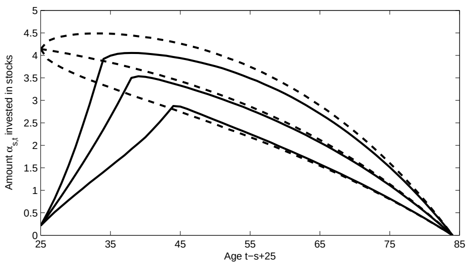
$$U_s \equiv \int_0^{60} e^{-\beta v} u\left(C^{ce}\right) dv, \tag{2.13}$$



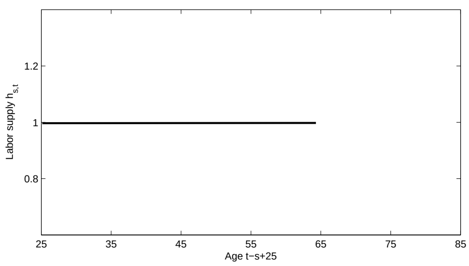
(a) Consumption $C_{s,t}$



(b) Financial wealth $F_{s,t}$



(c) Amount $\alpha_{s,t}$ invested in risky asset



(d) Labor supply $h_{s,t}$

Figure 2.1: *The 5%, 50% and 95% percentiles of the model variables in the autarky solution in absence (dashed lines) and in presence (solid lines) of a borrowing constraint. Calculations are based upon the default model parameters in Table 2.1. The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85. For the calculation under the borrowing constraint, labor-supply choices are assumed constant at level* h^*. *Notice that this is a simplification, because the borrowing constraint induces distortions in labor supply: the investor has an incentive to increase labor supply in order to accumulate more financial wealth to weaken the borrowing constraint.*

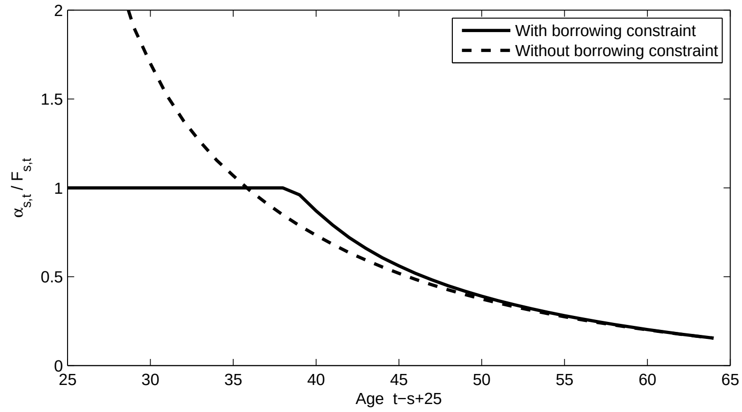


Figure 2.2: *The 50% quantile for the fraction $\alpha_{s,t}/F_{s,t}$ of financial wealth invested in stocks, with and without the borrowing constraint. Calculations correspond to the benchmark parameters contained in Table 2.1. The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85.*

where U_s is specified in equation (2.8). For the default parameters, the optimal consumption-investment strategy results in a certainty-equivalent consumption level of 0.8296. In absence of investments in the stock market (i.e. $\alpha_{s,t}$=0 for all s and all t), consumption is constant at a substantially lower level of 0.7870. Hence, the welfare gain from risky investments in the stock market is equal to (0.8296-0.7870)/0.7870=5.4%.[9] In absence of investments in the stock market, the investor is not able to take advantage of the risk premium in financial market, which reduces welfare.

Figure 2.2 has shown that for young investors the optimal amount $\alpha_{s,t}$ invested in stocks exceeds the financial wealth level $F_{s,t}$. The optimal solution thus requires the investor to be able to borrow against future labor-earnings when participat-

[9]This figure is in same order-of-magnitude as the 6.8% gain that was obtained in the two-agent setting of chapter 1. Recall that the parameters for the return on stocks was based upon a duration of investments of 30 years in the two-agent setting. This is roughly consistent with the duration of investments in this section: the average saving occurs in the middle of the working period at time $s + 20$ while the average dissaving takes place 30 years later in the middle of the retirement period at time $s + 50$.

ing in the stock market.[10] Human wealth typically cannot be used as collateral for stock investments by individual investors.[11] It is therefore imposed that the individual investor is subject to the borrowing constraint, i.e. he is subject to the additional condition:

$$F_{s,t} \geq 0 \tag{2.14}$$

for all s and all t.

The collective pension fund is able to outperform an individual investor because of two reasons: the pension fund is able invest in the financial market on behalf of unborn generations, thereby alleviating the *biological constraint* of financial markets that prevents unborn individuals from trading. Second, a collective pension fund is able to alleviate the *borrowing constraint* faced by young workers, who are unable to use their future labor earnings as collateral when trading in financial markets. A pension fund is able to alleviate both constraints: the power-of-taxation provides a pension fund with a claim on future labor earnings and enables it to invest in the stock market on behalf of young and unborn participants. Notice that it is natural to involve the borrowing constraint is the analysis, because both the *biological constraint* and the *borrowing constraint* are due to the

[10] I abstract from investment strategies that use financial derivatives such as call and put-options in the asset menu. Call-options on stocks enable young investors to have a leveraged position in stocks, i.e. attain a high equity exposure with a relatively small amount invested. Tan (2008) derives optimal portfolio strategies in the presence of derivatives in the asset menu. Notice, however, that equities are also leveraged assets in a sense, because equities provide a residual claim on profits after more secure interest payments have been paid to bond holders.

[11] The finding that young investors are borrowing constraint is a common feature of life-cycle investing models, see e.g. Heaton and Lucas (1997), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005) and Gomes, Kotlikoff, and Viceira (2008). The high demand for stocks by the young is robust to the inclusion of an empirically calibrated correlation between the innovations in labor earnings and stock returns, see Jagannathan and Kocherlakota (1996), Heaton and Lucas (1997), Cocco, Gomes, and Maenhout (2005). In addition, Heaton and Lucas (1997) show that the young remain borrowing constrained in the presence of significant transaction costs in the stock market. Constantinades, Donaldson, and Mehra (2002) are able to explain the equity-premium puzzle by using a model that restricts young workers from borrowing to finance investments in equity. Yet not all the literature concurs that young investors want to borrow against their future labor earnings to invest in stocks. In a model in which dividends and labor income are co-integrated, Benzoni, Collin-Dufresne, and Goldstein (2007) find that young investors like to have a short position in stocks in a model. The framework of Benzoni, Collin-Dufresne, and Goldstein (2007) is treated in chapter 4.

same underlying assumption, namely the assumption the pension fund is able to use future labor earnings as collateral when investing in the financial market.

The solid lines in Figure 2.1 illustrate the solution under theborrowing constraint. No known explicit solution exists for the intertemporal consumption-portfolio choice problem under a borrowing constraint.[12] The optimization problem is therefore solved via numerical solution techniques, using backward induction, state-space discretization, spline-interpolation and Gaussian quadrature. These techniques will also be used in chapter 3 and are explained in more detail in Appendix 3.A. Figure 2.1 illustrates that the borrowing constraint prevents young investors from capitalizing future labor income, and prevents investors from taking full advantage of the risk premium in financial markets. For the benchmark parameters, the borrowing constraint causes the certainty-equivalent consumption level to fall by 1.1%, from 0.8296 to 0.8208.

The borrowing constraint in equation (2.14) prevents an efficient allocation of risk between young and old generations as in equation (2.12). Under the borrowing constraint the exposure of young investors is lower than the risk exposure of the old. Hence, there is a role for a pension fund to redistribute risk to young generations. A social planner is able to collateralize future labor earnings, thereby alleviating the borrowing constraint of the young.

2.4 Risk Sharing

This section treats the case where risk-sharing is facilitated by a pension fund.

2.4.1 The pension fund policy

Recall that the overlapping generation framework features 40 + 20 overlapping generations in each period of time: 40 working and 20 retired generations. The pension fund collects contributions from the 40 working generations and pays out

[12] He and Pages (1993) provide an early treatment in context of the standard model of Samuelson (1969) and Merton (1969) in which they proof the existence of an optimal policy. Later contributions are provided by Cuoco (1997) and El Karouil and Jeanblanc-Picqu (1998). Grossman and Vila (1992) derive an explicit solution for the case of an infinite investment horizon.

benefits to the 20 retired generations. Individuals are not allowed to save, borrow or invest outside the pension fund, implying that consumption levels obey:[13]

$$C_{s,t} = \begin{cases} (1 - \pi_{s,t})\, wh_{s,t} & \text{if } t - s < 40 \\ b_{s,t} & \text{if } t - s \geq 40 \end{cases} \tag{2.15}$$

where $\pi_{s,t}$ and $b_{s,t}$ denote the contribution rate and the benefit level of cohort s at time t. Let the value of the financial assets of the pension fund at time t be denoted by F_t. The amount α_t invested in stocks by the pension fund at time t is specified as:

$$\alpha_t = \alpha(F_t), \tag{2.16}$$

where $\alpha(\cdot)$ is a time-independent policy function of the pension fund that governs the relationship between portfolio holdings and financial assets. The pension fund applies the same contribution rate for all workers:

$$\pi_{s,t} \equiv \pi_t = \pi(F_t), \tag{2.17}$$

where $\pi(\cdot)$ is a time-independent policy function of the pension fund that governs the relationship between the contribution rate and financial assets. The benefit formula is a function of pension-fund assets and past labor-supply levels:

$$b_{s,t} = b(F_t) \frac{\int_s^{s+40} e^{-r(v-s)} h_{s,v} dv}{\int_s^{s+40} e^{-r(v-s)} h^* dv}, \tag{2.18}$$

where $b(\cdot)$ is a time-independent policy function of the pension fund that governs the relationship between benefit levels and financial assets. The benefit formula is characterized by the following two properties. First, the rule simplifies into $b_{s,t} = b(F_t)$ if labor supply is undistorted or inelastic (i.e. if $h_{s,t} = h^*$ for all s and all t). As the second characteristic, supplying labor as a young worker results in more pension benefits than supplying labor when old. This feature of the benefit formula reflects the time-value of money: compared to labor supplied at age $u+1$, labor supplied at age u yields more pension benefits during retirement by a factor e^r.[14]

[13] The model features the property that individuals may have an incentive to save, borrow or invest outside the pension fund, see also footnote 17. In contrast, there is no such incentive in the model of chapter 3, in which the pension fund policy is fully optimized.

[14] The choice to use the riskfree rate as the measure of the time-value of money in the benefit specification is somewhat arbitrary. As an alternative, one can apply the unique discount factor that corresponds to the marginal utility of the individual.

The pension fund thus has three policy instruments to its disposal: $\alpha(\cdot)$, $\pi(\cdot)$ and $b(\cdot)$. The policy rules in equations (2.16)-(2.18) are relatively simple and are consistent with the common practice of pension funds around the world. There are two important ways, however, in which the policy specification is restrictive. First, the policy specification is restrictive because the rules with respect to contributions, benefits and investments are a function of pension fund assets F_t only, while past labor-supply choices are also state variables of the model as they determine future benefit levels. Second, contribution and benefit rules are *uniform*, in the sense that all workers pledge the same contribution rate and that relative adjustments in accumulated pension rights are the same for all participants.[15]

It is proven in section 2.4.3 that both restrictions are *not* binding in the special case where labor supply is inelastic: linear policy rules that are independent of age are optimal in the absence of labor-supply effects. Under elastic labor-supply, however, both restrictions become binding. Therefore, chapter 3 solves the policy rules of the pension fund without restrictions. This allows chapter 3 to evaluate how labor-market distortions are mitigated if restrictions on policy rules are relaxed.

Labor-supply choices $h_{s,t}$ solve from the first-order derivative of expected utility U_s with respect to labor-supply. Appendix 2.A shows that:

$$h_{s,t} = (w - w\pi_t + w\psi_{s,t})^{\epsilon} = h^*(1 - \pi_t + \psi_{s,t})^{\epsilon}, \tag{2.19}$$

where $w\psi_{s,t}$ is given by:[16]

$$w\psi_{s,t} = \int_{s+40}^{s+60} \mathbf{E}_t \left[e^{-\beta(v-t)} \frac{u'(C_{s,v})}{u'(C_{s,t}, h_{s,t})} \left(b(W_v) \frac{e^{-r(v-s)}}{\int_s^{s+40} e^{-r(w-s)} dw} \frac{1}{h^*} \right) \right] dv. \tag{2.20}$$

The term $w\psi_{s,t}$ denotes the utility value of accrued pension entitlements per unit of labor supply.[17] The term $b(W_v) \frac{e^{-r(v-s)}}{\int_s^{s+40} e^{-r(v-s)} dv} \frac{1}{h^*}$ in equation (2.20) represents

[15] It is stated in equation (2.17) that contribution rates are uniform. Furthermore, the uniformity of relative adjustments in benefit levels follows from equation (2.18): the value of previously accumulated pension rights are adjusted by $b(\Delta F_t)$ in response to a wealth shock ΔF_t (assuming the funding ratio remains unchanged from time t onwards.)

[16] In equation (2.20), $u'(C_{s,t}, h_{s,t}))$ is short notation for $\left(C_{s,t} - \frac{\epsilon}{\epsilon+1}\left(h_{s,t}\right)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}\left(h^*\right)^{\frac{\epsilon+1}{\epsilon}}\right)^{-\gamma}$

[17] In this chapter, the utility value of accruals does *not* correspond to the market value. This is due to the restrictions imposed on the pension contract. In chapter 3, in which the restrictions are relaxed, implying that the utility and market values of accruals coincide.

the increase in the benefit level at time v due to a unit-increase in labor supply at time t. The term $e^{-\beta(v-t)}\frac{u'(C_{s,v})}{u'(C_{s,t},h_{s,t})}$ is a 'subjective pricing-kernel' that gives the utility value at time t of a unit-increase in consumption at time v.

Equation (2.19) states that labor-supply choices are fully determined by 'effective marginal wage-rate' of a pension fund participant, i.e. the wage rate w minus pension contributions $w\pi_t$ plus the utility value of pension accruals $w\psi_{s,t}$. The pension fund distorts labor-supply choices if the contribution rate deviates from the accrual rate ($\pi_t \neq \psi_{s,t}$) and if labor supply is elastic ($\epsilon > 0$). A net tax is levied upon labor earnings if the contribution rate exceeds the accrual rate (i.e. $\pi_t > \psi_{s,t}$). A net subsidy is provided in the opposite case.[18] It follows from equation (2.19) that the indirect taxes and subsidies from the pension policy are proportional to labor earnings in the model. As explained in section 1.2.4 in chapter 1, proportional taxes and subsidies are, by approximation, optimal for the preferences of the model.

Finally, it needs to be specified how the pension fund is initialized. I follow Gollier (2008) by taking the perspective of a pension reform. The reform is implemented as follows. Suppose that we are in the autarky-situation of section 2.3, where there are 40+20 cohorts saving and investing on an individual retirement account under the borrowing constraint. At a certain point in time, normalized to t_0, all currently-living cohorts agree to transfer the wealth in their individual retirement accounts to a benevolent social-planner. The initial wealth F_{t_0} of the pension fund thus equals the sum of the wealth in the individual retirement accounts of the autarky investors at the time of the transition: $F_{t_0} = \sum_{s=t_0-59}^{t_0} F_{s,t_0}$, where F_{s,t_0} is defined in equation (2.9). The size of the wealth transfer F_{t_0} is stochastic, because it depends on the returns on investments of the autarky investors before t_0. There are thus many different possible scenarios for the reform. It is numerically too complex to evaluate all these different scenarios. Therefore, I follow Gollier (2008) by restricting the analysis to a single scenario for the reform. I focus on the scenario in which the amounts F_{s,t_0} in the individual retirement accounts are

[18]The model takes the perspective of rational economic agents. If pension fund participants suffer from myopia, i.e. if they heavily discount consumption the far-away future, a low utility value is attached to accrued pension rights. Under hyperbolic discounting, mandatory pension contributions therefore decrease the (subjective) effective wage rate against which labor is supplied, and hence discourage working, *even* if the contribution rate is actuarially fair.

equal to their certainty-equivalents.[19] For the benchmark parameters, it follows that $F_{t_0} = 388wh^*$, where wh^* represents annual undistorted labor-earnings.[20]

The total wealth of the social planner consists of the sum of financial wealth and the discounted value of the future labor earnings of all currently-living and unborn agents. If labor supply is inelastic, future labor earnings are constant over time so that human wealth equals

$$H = \sum_s H_{s,t} \tag{2.21}$$

where $H_{s,t} = \int_t^{s+40} e^{-r(v-t)} wh^* dv$ if $0 \leq t - s < 40$, $H_{s,t} = \int_s^{s+40} e^{-r(v-t)} wh^* dv$ if $t < s$ and zero otherwise. For the default parameters we have $H = 2040wh^*$. This number can be decomposed in two parts: the discounted value of the labor earnings of currently-living generations ($\sum_{s \leq t} H_{s,t} = 645wh^*$) and of unborn generations ($\sum_{s>t} H_{s,t} = 1395wh^*$). In terms of total wealth (e.g. the sum of financial and human wealth), the wealth of currently-living generations is approximately equal to the wealth of unborn ones at time t_0: $388 + 645 = 1033wh^*$ versus $1395wh^*$.[21]

[19] Formally, the certainty-equivalent wealth level F^{ce}_{s,t_0} is defined as the fixed amount that leaves the welfare level U^{aut}_s unchanged if it unexpectedly replaces the stochastic wealth level F_{s,t_0} of the autarky investor at time t_0.

[20] The initial value of pension fund assets is almost 9.7 times as large as the annual earnings of working participants, which are equal to $40wh^*$. This number of roughly consistent with real-life observations. For example, the ABP Pension Fund for Dutch civil servants had 216 billion Euro in assets at the end of 2007. During 2007, it received 6.7 billion in contributions while applying a contribution rate of 19%. The total wage earnings of participants were thus equal to 6.7/0.19=35 billion, implying that assets are 216/35=6.2 times labor earnings. The Dutch pension system roughly consists for 50% of a funded occupational pillar for 50% of a pay-as-you-go social security scheme (third-pillar private pension savings are relatively small in the Netherlands), implying that assets would have been 12.4 times annual labor earnings if the pension system were fully funded.

[21] Thereby, the benchmark parameters in this chapter are thus roughly consistent with the benchmark parameters in the two-agent model of chapter 1, in which both agents were assumed to be equally wealthy in discounted terms.

2.4.2 Optimization problem

The pension fund maximizes the aggregated utility of currently-living and unborn participants:

$$U = \max_{\alpha(\cdot),\pi(\cdot),b(\cdot)} \left\{ \mathbf{E}_{t_0} \left[\int_{t_0}^{\infty} e^{-\delta(t-t_0)} \left(\sum_{\Omega_t^{working}} u\left(C_{s,t}, h_{s,t}\right) + \sum_{\Omega_t^{retired}} u(C_{s,t}) \right) \right] dt \right\}, \tag{2.22}$$

subject to the budget constraints

$$\begin{aligned} dF_{t+1} &= rF_t dt + \alpha_t dX_t/X_t + \sum_{\Omega_t^{working}} \pi_t w h_{s,t} dt - \sum_{\Omega_t^{retired}} b_{s,t} dt, && (2.23a) \\ F_{t_0} &= 388wh^*, && (2.23b) \\ F_t &> -\frac{1}{\epsilon+1}\left(1-\frac{1}{\epsilon+1}\right)^{\epsilon} H \qquad \text{for all } t, && (2.23c) \end{aligned}$$

where $\Omega_t^{working}$ denotes the set of working cohorts at time t, i.e. $\Omega_t^{working} = \{s \in \mathbb{N} : \ t-40 < s \leq t\}$, and where $\Omega_t^{retired}$ denotes the set of retired cohorts at time t, i.e. $\Omega_t^{retired} = \{s \in \mathbb{N} : \ t-40 < s \leq t\}$. Parameter δ denotes the time-discount rate of the social planner. Following Gollier (2008), δ is determined such that all cohorts $s \geq t_0$ share equally in gain from risk sharing:

$$U_s = U_{s'} \qquad \forall s, s' > t_0 \tag{2.24}$$

where U_s is defined in equation (2.5).[22] The budget constraint in equation (2.23c) binds the value of pension fund assets from below: the pension fund cannot borrow more than the value of future labor earnings that can be collateralized. The constraint in equation (2.23c) is based upon the *maximum amount* of labor earnings that can be collateralized by the pension fund, which equals $\frac{1}{\epsilon+1}\left(1-\frac{1}{\epsilon+1}\right)^{\epsilon} H$. To see this, first notice that H is the amount of future earnings that can be collateralized by the pension fund if labor supply is inelastic (i.e. $\epsilon = 0$), in which the

[22] The restriction in equation (2.24) rules out any "precautionary savings" at the collective level. That is: the pension fund is not allowed to pay lower benefits to retirees in the short run in order to accumulate a buffer to reduce contribution rates (and hence reduce distortions in the labor market) in the long run. Equation (2.24) requires that currently-living generations benefit just as much from risk sharing as unborn generations, implying that such "precautionary savings" at the collective level are not allowed.

constraint in equation (2.23c) reduces into $F_t > -H$. On the other extreme, borrowing is not possible (i.e. $F_t \geq 0$) in a situation of infinitely-elastic labor-supply (i.e. if $\epsilon \to \infty$), because any distortion in the wage rate results in an infinite adjustment in labor supply. In general (i.e. $0 < \epsilon < \infty$), the pension fund faces a trade-off: a higher contribution rate (relative to the accrual rate) yields a higher fraction of earnings but also erodes the tax-base of the pension fund due to a reduction in labor supply. It follows from equation (2.19) that the wage differential x (i.e. the accrual rate minus the contribution rate) that maximizes revenues equals $\max_x \{xw\,(h^*(1-x)^\epsilon)\} = \frac{1}{\epsilon+1}$, which causes labor-supply to drop to $\left(1-\frac{1}{\epsilon+1}\right)^\epsilon$% of the undistorted level. As a result, the amount of future labor-earnings that can be collateralized by the pension fund equals $\frac{1}{\epsilon+1}\left(1-\frac{1}{\epsilon+1}\right)^\epsilon H$. As an example, if $\epsilon = 0.5$ the pension fund maximizes its revenues by applying a wage-differential of 66%, causing labor earnings to fall to 58% of the undistorted level. The pension fund is able to collateralize only $0.66 \times 0.58 = 38\%$ of undistorted future labor earnings H.

2.4.3 Special case: inelastic labor-supply

Under inelastic labor-supply (i.e. $\epsilon = 0$), preferences simplify into standard CRRA-utility over consumption. In addition, the benefit formula in equation (2.18) reduces into $b_{s,t} \equiv b_t = b(F_t)$. The optimization problem reduces into a version of the model close to Gollier (2008) and adopts an analytic solution, which is derived in Appendix 2.A.[23] The optimal solution features the property that consumption levels of workers and retirees are equal to each other:

$$(1-\pi_t)\,wh^* = b_t \equiv C_t, \tag{2.25}$$

for all t. The consumption rule in the optimal pension fund solution features consumption smoothing: unexpected wealth shocks from risk-taking are levied proportionally equally over all future time periods:

$$\frac{\partial C_v}{C_v} = \frac{\partial W_t}{W_t}, \tag{2.26}$$

for all $v > t$, where $W_t = F_t + H$. Accordingly, a drop in wealth by y% percent results in a decline in consumption in all future periods by y% percent, rather than

[23]The model is not identical to Gollier (2008), because the assumption of defined-contribution is not imposed.

being absorbed in a few periods. Thereby, the consumption rule in the risk sharing solution is consistent with equation (1.13) in the two-agent setting of chapter 1. In the optimal investment strategy, stock investments are equal to a constant fraction $\mu/(\gamma\sigma^2) = \lambda/(\gamma\sigma)$ of wealth:

$$\alpha_t = \frac{\lambda}{\gamma\sigma} W_t. \tag{2.27}$$

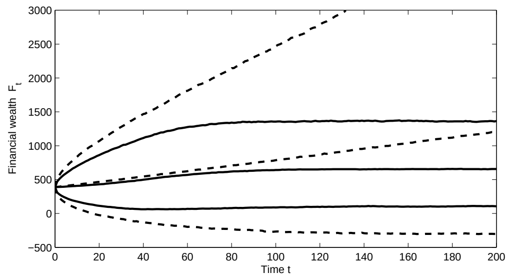
Thereby, the investment strategy of the pension fund is consistent with equation (1.14) in the two-agent setting of chapter 1. Comparing equations (2.11) and (2.27), it follows that risk sharing increases the demand for the risky asset. This was pointed out by Gollier (2008). The demand for stocks increases by the ratio of the wealth of unborn cohorts relative to the wealth of currently-living ones. For the default parameters, currently-living and unborn generations are approximately equally wealthy, implying that the demand for stocks roughly doubles at the date t_0 of the transition. At the initial time t_0, the pension fund invests $\alpha_{t_0}/F_{t_0} = 364/388 = 94.0\%$ of financial wealth in the stock. The remaining 6.0% is invested in the riskfree rate.

The dashed lines in Figure 2.3 show the optimal solution under inelastic labor-supply. The consumption-smoothing property causes adjustments in consumption to be permanent in nature, implying that consumption to follow a random-walk and hence diverges over time.

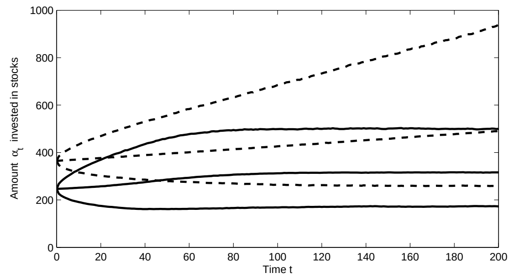
Under the optimal solution, the certainty-equivalent consumption level is equal to 0.8826, which corresponds to an increase of (0.8826-0.8208)/0.8208=7.5% in comparison to autarky.[24] The 7.5% gain from risk sharing can be decomposed in two components: (0.8826-0.8296)/0.8208=6.4% is due to risk sharing between non-overlapping generations, while the other (0.8296-0.8208)/0.8208=1.1% is due to the alleviation of the borrowing constraint. Both gains are possible due to the ability of the social planner to commit young and future generations to the risk-sharing contract. This allows the pension fund to use future labor earnings as collateral when investing in the stock market, implying that risk sharing enables individuals to take full advantage of the equity premium in the financial market.

The gain from risk sharing is very sensitive to the interest rate r. A lower interest rate causes the discounted value of the future labor earnings of unborn

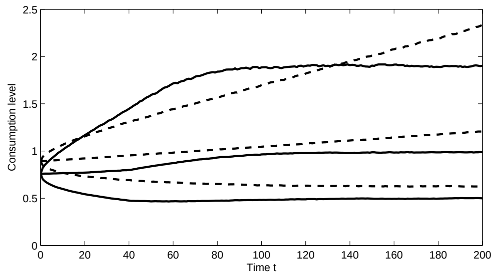
[24] This figure is in the same order-of-magnitude as the 6.5% that was found in the two-agent setting of chapter 1.



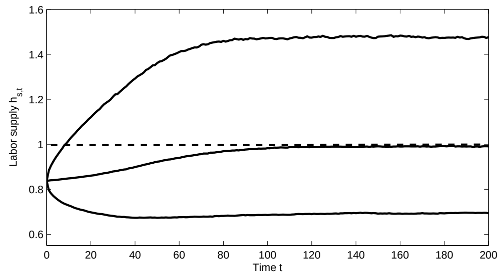
(a) Financial wealth F_t



(b) Amount α_t invested in stocks



(c) Consumption $C_{s,t}$



(d) Labor supply $h_{s,t}$

Figure 2.3: *The simulated 5%, 50% and 95% quantiles for pension fund assets W_t, the amount α_t invested in stocks, the consumption level and the labor-supply level. On the horizontal axis, the initial time t_0 is normalized to 0. The dotted lines refer to the case with inelastic labor supply ($\epsilon = 0$) while the solid lines refer to elastic labor supply ($\epsilon = 1$). For the case with elastic labor supply, the consumption represent the average over all cohorts, while effective wage rates and labor-supply levels represent the average over all working cohorts. Calculations are based upon the default parameters in Table 2.1.*

generations to become higher relative to the wealth of currently-living generations. As pointed out by the two-agent setting of chapter 1, the welfare gains from risk sharing are larger if unborn generations relatively wealthy. Indeed, lowering r from 2% to 1% increases the gain from risk sharing from 7.5% to 16.2%. Increasing r from 2% to 3% reduces the gain from risk sharing from 7.5% to 6.5%.

In a defined-contribution setting as in Gollier (2008), the expression for the optimal portfolio choice in equation (2.27) remains the same, except that H in the wealth definition $W_t = F_t + H$ is being replaced by the discounted value of future *savings* instead of future labor earnings. Given that savings a fraction of earnings, the risk bearing capacity is substantially reduced. Due to the reduction in the risk bearing capacity, the gain from risk sharing is smaller in a defined contribution setting. For the benchmark parameters, the gain from risk sharing reduces by more than half: from 7.5% to 3.4%.[25]

2.4.4 General case: elastic labor-supply

Under elastic labor supply, the optimization problem does not adopt an analytical solution. The problem is therefore solved via numerical solution-techniques. The numerical method is described in detail in Appendix 2.A. In contrast to chapters 3 and 4, the numerical method is *not* based upon backward-induction techniques because the number of state variables is extremely large: all past labor-supply choices of currently-living agents are (endogenous) state variables of the model because they are informative about future benefit levels. The optimization problem in this chapter is therefore solved via forward-running Monte-Carlo simulations. A Taylor expansion is applied to the policy-functions $\pi(\cdot)$, $b(\cdot)$ and $\alpha(\cdot)$, of which the coefficients are solved via a grid-search algorithm. The time-discount rate δ of the social planner solves endogenously from the model via equation (2.24). The value $\psi_{s,t}$ of pension accruals is obtained via across-path regressions, which is possible because the expression of $\psi_{s,t}$ in equation (2.20) takes the form of a conditional expectation. Across-path regressions have been applied by Longstaff and Schwartz (2001) in the context of option pricing and by Brandt, Goyal, Santa-Clara, and

[25] The welfare analysis under the defined contributions restrictions is executed along the lines of Gollier (2008), in which utility is specified over retirement consumption levels only. If utility function also recognizes consumption levels during the working period (which are not affected by risk sharing), the defined-contributions assumptions has an even larger impact on welfare.

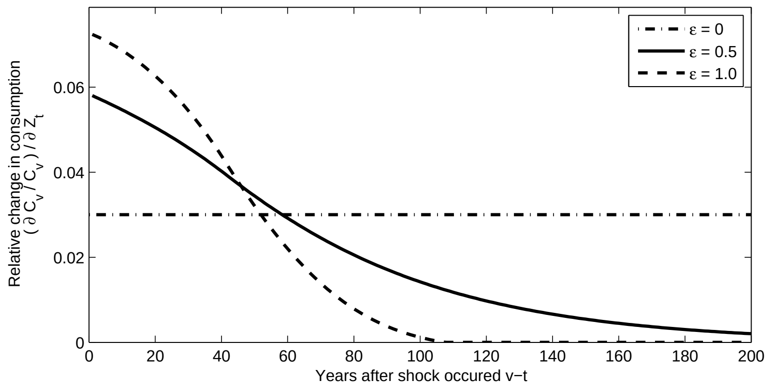


Figure 2.4: *The elasticity of consumption C_v at times $v > t$ in response to an economic shock Z_t at time t: $(\partial C_v/C_v)/(\partial Z_t)$. For the case with elastic labor-supply, the reported elasticities correspond to the unconditional average. Calculations are based upon the default parameters in Table 2.1.*

Stroud (2005) and Koijen, Nijman, and Werker (2009) in the context of dynamic consumption-portfolio choice.

The solid lines in Figure 2.3 show the optimal solution under elastic labor-supply. In contrast to the case with inelastic labor-supply, the model variables do not follow a random walk anymore. Instead, the model variables adopt a stationary distribution in the long run.[26] Hence, shocks are not permanent in nature anymore, but are recouped within a certain time-period. The model thus does not feature perfect consumption smoothing anymore. This is due to the following reason. Consumption smoothing implies that labor-market distortions are smoothed over time. That is, every shock induces a permanent distortion in the labor market. Hence, the distortions from a current shock add to the distortions from *all* previous shocks that have occurred in the past. It is well-known that the marginal costs from distortions are higher if they add to already-existing distortions from shocks in the past. Hence, the welfare costs from distortions become very large in the long run if consumption smoothing is applied.

[26]To arrive at a stationary solution, the pension fund adjusts the level- and slope-coefficients of the policy functions $\alpha(\cdot)$, $\pi(\cdot)$ and $b(\cdot)$. The sign of the slope-coefficient of all three policy functions remains unchanged, because a change of sign destabilizes the solution.

Hence, in the presence of distortions in the labor market, the pension fund faces a trade-off between consumption-smoothing on the one hand and the reduction of labor-market distortions on the other hand. This trade-off is illustrated by the solid line in Figure 2.4. Under inelastic labor supply, the solution features consumption smoothing: all future consumption levels are equally elastic with respect to an economic shock at present. In the presence of distortions in labor supply, nearby consumption levels become more elastic with respect to a current shock than far-away consumption levels.[27] A financial shock is not smoothed over as many generations possible, but is levied primarily upon currently-living generations. Thereby, the solution is consistent the expression in equation 1.29 in the two-agent model of chapter 1.

Under elastic labor supply, it optimal to *recover* from financial shocks, instead of smoothing shocks over as many generations as possible. Thereby, the solution is consistent with solvency rules imposed by regulators that require a pension fund to recover from its losses in a relatively short time-period. As an example, the Dutch regulator requires pension funds with a funding deficit to recover within three years, and to have their financial buffer recovered within 15 years.

At first sight, the optimal solution appears to be inconsistent with the literature on tax-smoothing, which finds that governments should set their debt policies such that distortionary taxes are smoothed over time, see e.g. Barro (1979). Indeed, Figure 2.4 shows that there is no tax-smoothing from the perspective of a *single* shock. The solution in this section, however, is consistent with tax-smoothing in terms of all shocks *together*: the bandwidth of the confidence intervals of the effective wage rate are stationary over time in Figure 2.3. Hence, in order to accomplish tax-smoothing, current generations should take a disproportionally large stake in current shocks. If all shocks would be smoothed over all future periods, then later-born cohorts would share in more shocks than earlier-born ones, which is inconsistent with tax-smoothing.

Consistent with the result in equation (1.28) in the two-agent setting of chapter 1, labor supply effects reduce the demand for stocks. Table 2.2 shows that, for the benchmark parameters, the fraction of financial assets invested in stocks drops

[27]Thereby, labor-supply effects work in the opposite direction as (internal) habit formation. Habit-formation causes large drops in consumption to be unattractive, so that nearby consumption is *less* elastic with respect to a current shock than far-away consumption.

labor-supply elasticity (ϵ)	fraction of pension fund assets invested in stocks
0	94.0%
0.5	87.2%
1.0	63.5%

Table 2.2: *The initial fraction α_{t_0}/F_{t_0} of financial assets invested in stocks by the pension fund at time t_0, for various parameter choices of the elasticity of labor supply ϵ. Calculations are based upon the benchmark parameters in Table 2.1.*

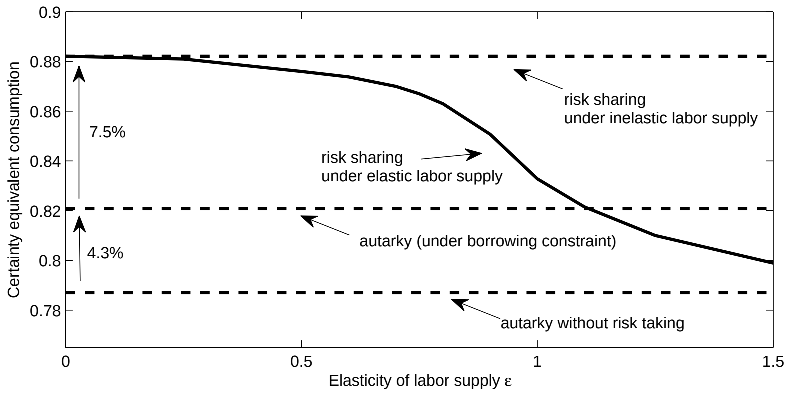


Figure 2.5: *The certainty-equivalent consumption level in the pension fund as a function of the wage-elasticity of labor supply ϵ. Calculations are based upon the default parameters as contained in Table 2.1.*

from 94% to 87.2% if the elasticity of labor supply increases from 0 to 0.5. If the elasticity of labor supply further increases to 1.0, the fraction of assets invested in stocks drops to 63.5%. The decline in the demand for stocks has two reasons. First, risk taking is less attractive if it is accompanied by labor-market distortions. As the second-order effect, substitution-elasticity in labor supply causes labor supply to become pro-cyclical: the effective wage rate of a pension fund participant decreases after a negative wealth shock (if labor earnings are taxed), and increases after a positive shock. Labor supply thus to becomes *more positively correlated to stock returns*, which reduces the appetite for risk taking in the investment portfolio.

The solid line in Figure 2.5 illustrates the welfare effects of distortions in the labor market. As reported in the previous section, the gain from risk sharing is equal to 7.5% in absence of labor supply effects (i.e. if $\epsilon = 0$). If the elasticity of labor supply increases from 0 to 0.5, labor supply effects have minor implications: only a small fraction of (0.8826-0.8760)/(0.8826-0.8208)=10.7% of the welfare gain from risk sharing is eroded by distortions. However, for larger values of ϵ, the welfare costs from distortions rapidly increase. If the elasticity of labor supply is 1, the majority of the gains from risk sharing is eroded: (0.8826-0.8328)/(0.8826-0.8208)=80.6%.

Under elastic labor supply, the pension fund is able to use the future labor earnings of young and unborn generations as collateral when investing in the stock market. However, in contrast to the analysis in section 2.4.3, making commitments on behalf of young and unborn generations comes at a cost. Young and unborn generations reduce their their labor supply ex-post in order to (partially) get around high contribution rates, or increase their labor supply to take advantage of low contribution rates. Thereby, labor-market distortions make it less attractive to use human wealth as collateral when investing in the stock market, thereby reducing the gains from risk sharing.

The risk sharing solution in this chapter features the property that there exists no Pareto-efficient risk-sharing solution if the elasticity of labor-supply exceeds the value of 1.1. This result is due to the restrictions that are imposed upon the policy rules, which prevent the social planner from replicating the autarky-solution. In the knife-edge case where $\epsilon = 1.1$, the gains from risk sharing are exactly offset by the costs from labor-market distortions.

In the limit where labor-supply becomes infinitely elastic (i.e. $\epsilon \to \infty$), any distortion to the wage rate results in an infinite response in labor-supply choices. In this situation, the contribution rate can never deviate from the value of pension rights accrued in return. Under the policy rules of this chapter, this implies that the pension fund is unable to invest in the stock market. Hence, in the limit where labor-supply becomes infinitely elastic, welfare converges to the autarky welfare-level in *absence* of risk-taking.

The large welfare losses due to labor-market distortions may be a result of the policy restrictions that have been imposed in this chapter. Therefore, chapter 3 examines to what extent labor-market distortions can be mitigated by relaxing these restrictions.

2.5 Conclusion

The analysis in this paper has shown that distortions in the labor market can prevent a pension fund from taking advantage of the gains from intergenerational risk sharing. To reduce the welfare costs from labor market distortions, it is optimal for a pension fund to recoup financial gains and losses from risk-taking primarily upon currently-living generations. The analysis thereby provides an economic justification for solvency regulations that require pension funds to recover from financial shortfalls in a relatively short period of time. Solvency regulations cannot be justified from the existing literature on intergenerational risk sharing, which finds that shocks should be smoothed over as many generations as possible. Smoothing shocks over many generations is not optimal anymore once the welfare costs from distortions in labor markets are recognized.

The analysis has also shown that the welfare effects from labor-supply flexility are not unambiguous. Labor-supply flexibility makes it more difficult for governments to commit future generations to share in current financial risks. The analysis in this paper therefore suggests that governments may be able to improve overall welfare if they are better able to tie workers to the insurance pool of the pension fund. This can be accomplished, for example, by restricting the portability of pension rights, or by sustaining implicit labor contracts that involve deferred wages and hence reduce labor-supply flexibility. In general, such policy proposals are in general considered to be welfare reducing, as they hurt the efficiency of the labor market. However, this chapter points out that the welfare effects of such proposals are mixed, because they improve the ability of pension funds to commit young and future generations to a risk sharing contract.

In further research, the results in this chapter can be extended to the context of public debt policies. Existing studies, e.g. Barro (1979), find that government debt policies should be set such that distortionary transfers are smoothed over time. The analysis in chapter 2 suggest that this can only be accomplished if current shocks are levied primarily upon current generations. Hence, an expansion of public debt should be followed by a period with budget surpluses to reduce government debt levels in a relatively short time-period. Given the large expansion of government debt observed in recent years, this result would be highly relevant for today's fiscal policy makers.

2.A Appendix

Proof of equations (2.10) **and** (2.11)

The Hamilton-Jacobi-Bellman equation for the optimization problem in autarky is given by

$$\beta J = \max_{\alpha, C} \left\{ \frac{C^{1-\gamma}}{1-\gamma} + J_W \left(rW + \sigma\lambda\alpha - C\right) + \frac{1}{2} J_{WW} \left(\alpha\sigma\right)^2 + J_t \right\}, \tag{2.28}$$

where the subscripts of the value function J denote first- and second order derivatives with respect to total wealth W_t and where the subscripts for cohort s and time t are omitted in the consumption and portfolio choices $C_{s,t}$ and $\alpha_{s,t}$. The value function takes the form

$$J = \frac{W^{1-\gamma}}{1-\gamma} g(60+s-t)^{\gamma}. \tag{2.29}$$

Substituting of the value function into equation (2.28) and solving for the resulting first-order conditions yields the optimal consumption and portfolio policy:

$$C_{s,t} = g(60+s-t)^{-1} W_{s,t}, \tag{2.30}$$

$$\alpha_{s,t} = \frac{\lambda}{\gamma\sigma} W_{s,t}. \tag{2.31}$$

Substitution of the optimal policies (2.30) and (2.31) into (2.28) provides a linear ordinary differential equation for the function $g(\dot{})$:

$$-\theta g(60+s-t) = 1 + \dot{g}(60+s-t), \tag{2.32}$$

where

$$\theta = \frac{(1-\gamma)r - \beta}{\gamma} + \frac{1}{2}\frac{1-\gamma}{\gamma^2}\lambda^2. \tag{2.33}$$

Together with the requirement that $g(s+60) = 0$, it follows that:

$$g(60+s-t) = \frac{1}{\theta}\left(e^{\theta(60+s-t)} - 1\right), \tag{2.34}$$

Applying Ito's lemma to (2.30), it is obtained that the consumption dynamics are given by:

$$dC_{s,t} = g(60+s-t)^{-1}dW_{s,t} - \frac{\dot{g}(60+s-t)W_{s,t}}{g(60-t+s)^2}dt. \tag{2.35}$$

Employing the optimal portfolio rule given in (2.31), the wealth dynamics are given by:

$$\frac{dW_{s,t}}{W_{s,t}} = \left(\frac{\lambda^2}{\gamma} + r - g(60-t+s)^{-1}\right)dt + \frac{\lambda}{\gamma}dZ_t. \tag{2.36}$$

Using this expression for $g(60+s-t)$ in equation (2.34), we have

$$-g(60+s-t) - \frac{\dot{g}(60+s-t)}{g(60+s-t)} = \theta. \tag{2.37}$$

Substitution the results in equations (2.36) and (2.51) into (2.35), we establish:

$$\frac{dC_{s,t}}{C_{s,t}} = \left(\frac{\lambda^2}{\gamma} + r + A\right)dt + \frac{\lambda}{\gamma}dZ_t. \tag{2.38}$$

It follows from equation (2.30) that

$$\partial C_{s,t}/C_{s,t} = \partial W_{s,t}/W_{s,t}, \tag{2.39}$$

since $g(\cdot)$ is a deterministic function of time. From equation (2.38) it follows that consumption growth is independent of wealth such that:

$$\partial C_{s,u}/C_{s,u} = \partial C_{s,t}/C_{s,t}, \tag{2.40}$$

for all $u > t$. Combining equations (2.39) and (2.40) yields the consumption smoothing property:

$$\partial C_{s,u}/C_{s,u} = \partial W_{s,t}/W_{s,t}, \tag{2.41}$$

for all $u > t$.

Proof of equation (2.19)

For an individual who entered the pension fund at time s, the partial derivative of utility U_s with respect to labor supply $h_{s,t}$ is given by:

$$0 = \frac{\partial \mathbf{E_t}\left[\int_s^{s+40} e^{-\beta(v-s)}u\left(C_{s,v}, h_{s,v}\right)dv + \int_{s+40}^{s+60} e^{-\beta(v-s)}u\left(C_{s,v}\right)dv\right]}{\partial h_{s,t}}. \tag{2.42}$$

Observing that labor supply at time t only affects utility from consumption and leisure at time t and utility gained from consumption during retirement, the first-order-condition simplifies into:

$$0 = e^{-\beta(t-s)} \frac{\partial u\left(C_{s,t}, h_{s,t}\right)}{\partial h_{s,t}} + \frac{\partial \mathbf{E}_t \left[\int_{s+40}^{s+60} e^{-\beta(v-s)} u(C_{s,v}) dv\right]}{\partial h_{s,t}}. \tag{2.43}$$

The first term on the right-hand-side of equation (2.43) is rewritten as:

$$\begin{aligned} & e^{-\beta(t-s)} \frac{\partial u\left(C_{s,t}, h_{s,t}\right)}{\partial h_{s,t}} \\ = \quad & e^{-\beta(t-s)} \left(C_{s,t} - \frac{\epsilon}{\epsilon+1}\left(h_{s,t}\right)^{\frac{\epsilon+1}{\epsilon}} + \frac{\epsilon}{\epsilon+1}\left(h^*\right)^{\frac{\epsilon+1}{\epsilon}}\right)^{-\gamma} \left((1-\pi_t)w - \left(h_{s,t}\right)^{\frac{1}{\epsilon}}\right) \end{aligned} \tag{2.44}$$

Substitution of equation (2.18) allows us to rewrite the second term in equation (2.43) as:

$$\begin{aligned} & \frac{\partial \mathbf{E}_t \left[\int_{s+40}^{s+60} e^{-\beta(v-s)} u(C_{s,v})\right]}{\partial h_{s,t}} \\ = \quad & \mathbf{E}_t \left[\sum_{s+40}^{s+60} e^{-\beta(v-s)} \left(C_{s,v}\right)^{-\gamma} b(W_{s+v}) \frac{1}{h^*} \frac{e^{-r(v-s)}}{\int_s^{s+40} e^{-r(w-s)} dw} dv\right] \end{aligned} \tag{2.45}$$

Substitution of equations (2.44) and (2.45) into equation (2.43) and rewriting yields equation (2.19) and (2.20).

Proof of equations (2.25) **and** (2.27)

The Hamilton-Jacob-Bellman equation for the optimization problem in autarky is given by

$$\delta J = \max_{\alpha, C} \left\{60 \frac{C^{1-\gamma}}{1-\gamma} + J_W \left(rW + \sigma\lambda\alpha - 60C\right) + \frac{1}{2} J_{WW} \left(\alpha\sigma\right)^2 + J_t\right\}, \tag{2.46}$$

where the subscripts of the value function J denote first- and second order derivatives where the subscripts for cohort s and time t are omitted in the consumption variable $C_{s,t}$ and the portfolio choice $\alpha_{s,t}$. Similar to the optimization problem of the autarky-investor, the value function takes the form

$$J = \frac{W^{1-\gamma}}{1-\gamma} g^{\gamma}, \tag{2.47}$$

where g is now a constant. Substituting of the value function into equation (2.28) and solving for the resulting first-order conditions yields the optimal consumption and portfolio policy:

$$C_{s,t} = g^{-1} W_{s,t}, \tag{2.48}$$

$$\alpha_{s,t} = \frac{\lambda}{\gamma\sigma} W_{s,t}, \tag{2.49}$$

Substitution of the optimal policies (2.48) and (2.49) into (2.46) solves g:

$$\theta = \frac{60}{-g}, \tag{2.50}$$

where θ is defined in equation (2.33). Employing the optimal portfolio rule given in (2.49), the wealth dynamics are given by:

$$\frac{dW_{s,t}}{W_{s,t}} = \left(\frac{\lambda^2}{\gamma} + r + \theta\right) dt + \frac{\lambda}{\gamma} dZ_t. \tag{2.51}$$

Combining equations (2.48) and (2.51), it follows that:

$$\frac{dC_{s,t}}{C_{s,t}} = \left(\frac{\lambda^2}{\gamma} + r + \theta\right) dt + \frac{\lambda}{\gamma} dZ_t. \tag{2.52}$$

Combining equations (2.51) and (2.52) yields the consumption smoothing property:

$$\partial C_{s,u}/C_{s,u} = \partial W_{s,t}/W_{s,t}, \tag{2.53}$$

for all $u > t$.

Description of Numerical Method

This section describes the numerical solution method that is used for the calculations in section 2.4.4.

The three policy functions $pi(\cdot)$, $b(\cdot)$ and $\alpha(\cdot)$ are approximated by time-invariant first-order polynomials in F_t: $\pi(F_t) \approx \pi_0 + \pi_1 F_t$, $b(F_t) \approx b_0 + b_1 F_t$, $\alpha(F_t) \approx \alpha_0 + \alpha_1 F_t$. These approximations for the decision rules of the pension fund reduce the number of decision variables of the decision making problem to six scalars which need to be solved: π_0, π_1, b_0, b_1, α_0, α_1. These six parameters are solved by running Monte-Carlo simulations repeatedly. At the beginning of every new simulation run, the six parameters are adjusted according to a grid-search algorithm until the objective function of the social planner is maximized. All paths start are initiated at time t_0 where the pension reform takes place.

Recall that it is imposed in section 2.4.4 that parameter δ is chosen such that all generations are equally well off. This criterion is met by requiring the ex-ante welfare of earlier-born cohorts to be the same as later-born ones. Numerically, this is achieved by imposing the welfare of initial cohorts to be the same as those in the steady-state. The pension fund reaches its steady state after 100-150 years if the parameter of labor-supply elasticity is not too close to zero. This procedure for the determination of δ pins down any of the six unknown policy parameters as a function of the other five. As a result, the number of policy parameters that need to be solved reduces from six to five: the sixth decision variable is adjusted at the beginning of every new simulation run until the welfare levels of the initial and the steady state generation converge to each other.

The calculation of the labor-supply choice in equation (2.19) is based upon the utility value of pension accruals $\psi_{s,t}$. Notice from equation (2.20) that $\psi_{s,t}$ takes the form of a *conditional expectation*. Since simulation paths run *forward* in time, $\psi_{s,t}$ cannot be determined 'on the fly' on the basis of the current simulation run. Therefore, $\psi_{s,t}$ is derived by using the information of the previous simulation run (recall that simulation runs are run repeatedly). Given that we are working with a large number of simulations, the conditional expectations can be calculated on the basis of *across*-path regressions. In these regressions, the value of pension accruals $\psi_{s,t}$ is regressed against pension fund assets F_t and the square of pension fund assets $(F_t)^2$. It turns out that a second-order polynominal in F_t captures the variation in the value of pension accruals well. Simulation runs are repeated until the regression-coefficients of the across-path regressions converge.

Chapter 3

The Welfare Gains of Age-Differentiation in Pension Funds

Consistent with real-world practices, chapter 2 assumed the policy rules of the pension fund to be relatively simple. In particular, it was imposed in chapter 2 that contribution and benefit rules are *uniform* in age, in the sense that all workers pledge the same contribution rate and that relative adjustments in accumulated pension rights are the same for all participants. Thereby, chapter 2 provided valuable insights in the role of labor-market distortions in real-world environments. In the current chapter, the policy rules of the pension fund are fully optimized. Thereby, it is examined to what extent distortions in the labor market can be mitigated if age-specific policy rules are applied.

It is shown that age-specific policy rules are very effective in mitigating distortions in the labor market. The optimal solution strategy is characterized by the property that distortionary taxation of labor earnings is applied to unborn and young generations, for which the social planner lacks information about the future earnings capacities within cohorts. It is assumed that previous labor earnings are informative about future labor earnings, so that the social planner is able to apply non-distortionary taxation to older workers. The tangible assets of older workers are informative about previous labor earnings, so that financial shocks can be levied upon the participant in a non-distortionary way via the taxation of capital. The optimal solution is characterized by age-differentiation: the value of pension entitlements of the young is more responsive to economic shocks than the entitlements of the old.

3.1 Introduction

It is commonly observed that pension funds apply fairly simple policy rules with respect to contribution rates and benefit payments. Policy rules are typically *uniform* in age, in the sense that contribution rates and relative adjustments in accumulated pension rights are the same for all cohorts. This is also what has been imposed in the previous chapter. The current chapter studies the extent to which welfare can be improved if age-specific policy rules are applied, in which contribution rates and relative adjustments in accumulated pension rights are differentiated among cohorts. Under *age-specific* policy rules, contribution rates and relative adjustments in the value of accumulated pension rights are determined separately for each age-group in the pension fund, instead of at the aggregate level.

If labor-market distortions are absent, the answer to this research question is easy. Section 2.4.3 has showed that the uniformity-restriction is *not* binding if labor supply is inelastic. Under inelastic labor supply, the sole concern of a social planner is to maximize the utility over consumption. Uniform contribution and benefit rules are perfectly capable of implementing the first-best solution, in which shocks are smoothed proportionally equally over all future consumption levels of all generations (consumption smoothing). Hence, uniform policy rules are not constraining in terms of implementing consumption smoothing. However, uniform policy-rules are not optimal anymore if a pension fund is also concerned with distortions in the labor-market. An optimal pension contract uses distortionary transfers only in cases where lump-sum transfers are not possible.

This chapter shows that it is optimal for a pension fund, when recouping shocks via adjustments in the retirement income of its participants, to rely as much as possible on adjustments in *previously* accumulated pension rights, instead of adjustments in *future* pension accruals. Adjustments in future pension accruals distort future labor-supply choices: adjustments in future accruals induce substitution effects in labor supply, because lower (higher) accrual rates discourage (encourage) work.. Adjustments in previously accumulated pension rights, on the other hand, do *not* distort future labor-supply choices. Nor is there an anticipatory effect on savings or labor supply: future adjustments to pension rights can be both positive or negative, depending on future economic developments, implying that in expectation future adjustments are zero and anticipatory effects are absent.

The non-distortionary character of adjustments to accumulated wealth in the context of a pension scheme stands in sharp contrast to the situation of the government. Individuals typically anticipate that accumulated wealth will be taxed by the government, and are thus discouraged to accumulate wealth, thereby reducing their labor supply or reducing their savings level. The context of a pension fund, however, is different. A pension fund uses taxes and subsidies to recoup economic shocks upon participants. In roughly half the cases, namely if previous returns were below expectations, the pension fund levies a tax on the accumulated pension wealth of its participants, while in the other half of the cases, if previous returns were above expectations, the pension fund provides a subsidy. In expectation, there is thus roughly a zero tax on wealth in the future. As a result, there is thus no incentive for individuals to reducing their savings level or by reducing their labor supply ex-ante in anticipation of future adjustments to their pension wealth.

The analysis in this chapter takes the perspective where the pension fund provides full information on its future policy rules. Moreover, these policy rules are assumed to be credible, in the sense that the pension fund does not deviate from the policy rules that are announced beforehand. Participants can thus perfectly predict the future behavior of the pension fund. The optimal solution strategy is derived under the assumption that individuals optimize their labor supply behavior with respect to the policy rules of the pension fund, while the pension fund optimizes its policy rules by taking into account the labor supply responses of its participants.

The solution is not required to be time-consistent. That is: the pension fund may have an incentive to deviate from the rules that it has announced beforehand. For example, in a period of underfunding there is an incentive to reduce distortions in the labor market by cutting pension entitlements and reduce contribution rates. Arguably, a time-inconsistent solution strategy is therefore not realistic: the policy rules that are announced by pension fund may not be perceived as being credible by participants. On the other hand, it can be argued that a time-inconsistent solution is not problematic in the case where the contract is complete and legally enforceable. Thereby, a time-inconsistent solution provides valuable insights in the way in which the pension system can be improved if contracts are made complete and legally enforceable: the associated benefits can be measured by dropping the time-consistency requirement in the optimization problem.

The contribution of this chapter is fourfold. First, I find that age-specific policy rules are very effective in mitigating distortions in the labor market. Using the same benchmark parameters as in chapter 2, and assuming the elasticity of labor-supply equal to 1, I find that labor-market distortions erode only 18.0% of the gain from risk sharing under age-specific policy rules, whereas this is as much as 80.6% under uniform policy rules (derived in chapter 2). Hence, the welfare costs from labor-market distortions is dramatically reduced by the ability to differentiate the policy rules of the pension fund with respect to cohorts.

Second, I show that the optimal policy features the property that the value of pension rights of young workers is a lot more responsive to economic shocks than the pension rights of old workers and retirees. The results in this chapter therefore strengthen the case for age-differentiation in collective pension funds, as promoted in Molenaar, Munsters, and Ponds (2008).[1] In practice, this would imply that downward adjustments in value of pension rights as a result of a negative economic shock (often referred to as "cuts" in pension rights) are larger for young workers in comparison to old workers. At the same time, upward adjustments in the value of pension rights in response to a positive economic shock (often referred to as "inflation corrections" or "indexation") are also larger for young workers. The intuition for this result is that the wealth of young workers primarily consists of future labor earnings. As explained in the introduction of chapter 2, lump-sum transfers cannot be applied when future labor earnings are used as col-

[1] Molenaar, Munsters, and Ponds (2008) propose age-differentiation in collective pension funds to increase the risk-bearing capacity of young workers. The model of Molenaar, Munsters, and Ponds (2008) imposes accrual rates to be constant. Under this restriction, the pension fund is able to adjust the (future) retirement consumption levels of workers *only* via adjustments in previously accumulated pension rights. Given that the amount of pension rights accumulated by young workers is relatively low, Molenaar, Munsters, and Ponds (2008) find that the value of pension rights of young workers should be more responsive to economic shocks in comparison to the pension rights of the old.

lateral. Hence, to mitigate distortions in the labor market, the pension fund finds it is optimal to use previously accumulated pension rights as collateral whenever possible. Given that the amount of pension rights accumulated by young workers is small, the value of pension rights of young workers becomes very responsive to economic shocks in the optimal solution.

As the third contribution, I compare two methods that are used in the literature for the evaluation of risk sharing. Most studies take the perspective of a social planner who maximizes the aggregated utility of all cohorts. This approach, used e.g. in Gollier (2008), has been applied in chapter 2. Two recent studies, Teulings and De Vries (2006) and Ball and Mankiw (2007), have put forward an alternative approach, in which non-overlapping generations are able to trade risk with each other in a fictitious financial market. Section 1.2.3 has explained why this approach is able to measure the gains from risk sharing: if future generations are able to trade with currently-living generations, then risk can be shared between non-overlapping generations. Essentially, the method allows investors to participate in the financial market before birth (or before the date of labor-market entry). I show that these two methods differ from each other in an important way, namely in terms of their criterion for intergenerational fairness. In the approach of Teulings and De Vries (2006) and Ball and Mankiw (2007), generations are treated equal in terms of *market value*, because the risk-sharing solution is derived on the basis of market trading. A social planner, in contrast, is more flexible with respect to the welfare criterion that is applied. The study of Gollier (2008), for example, imposes that all generations are treated equal in terms of *ex-ante welfare levels*.[2]

This chapter points out that different criteria for intergenerational fairness have very different redistributional effects in ex-ante terms. I show that treating cohorts *equal* in terms of market value implies that cohorts are treated *unequal* in terms of ex-ante welfare levels: later-born cohorts benefit more from risk sharing than earlier-born ones. Vice versa, treating cohorts *equal* in welfare-terms implies that cohorts are treated *unequal* in terms market-value: the market-value of participation in the pension fund is positive for earlier-born cohorts while being negative for later-born ones. The quantitative difference between the two criteria for intergenerational fairness is large. For a cohort that enters at the time of ini-

[2] Equal treatment in ex-ante welfare terms has also been applied in chapter 2, see equation (2.24).

tiation of the pension fund, the difference is as large as a one-time bonus of 1.57 times annual earnings for the benchmark parameters.

The structure of the paper is as follows. Section 3.2 introduces the model for the pension fund. Section 3.3 derives the optimal policy of the pension fund. Section 3.4 introduces pre-labor-market-entry investments in stocks to the model. Section 3.5 compares the model to the model that was used in chapter 2. Finally, section 3.6 concludes.

3.2 The model

The specification for the economy, overlapping generations and individual preferences is the same as in chapter 2. The model for the policy of the pension fund, however, is optimized at the age-level instead of the aggregate level. As a result, the policy rules derived in this chapter are differentiated with respect to age, in contrast to the uniform policy that was derived in chapter 2. Consistent with chapter 2, the pension fund is initialized via a pension reform at time t_0. The model specification in this section describes the pension policy for all cohorts $s \in \mathbb{N} : s \geq t_0$ that enter the pension fund after the time t_0 of the reform. The policy for the initial cohorts $s \in \mathbb{N} : t_0 - n - m < s < t_0$, which are alive during the transition, will be clarified in section 3.5.

The pension fund

In contrast to chapter 2, the pension-fund model features the property that the participant does *not* have an incentive to save, borrow or invest outside the pension fund. As a result, the consumption level $C_{s,t}$ of an individual in cohort s at age t obeys:

$$C_{s,t} = \begin{cases} (1 - \pi_{s,t})h_{s,t}w & \text{if } s \leq t < s + 40 \\ b_{s,t} & \text{if } s + 40 \leq t \leq s + 60 \end{cases} \tag{3.1}$$

where $\pi_{s,t}$ denotes the contribution rate of workers and where $b_{s,t}$ denotes the benefit level of retirees. For each cohort s, the pension fund keeps track of two financial accounts: $A_{s,t}$ and $S_{s.t}$. The account $A_{s,t}$ denotes the value of *previously* *accumulated* pension rights, for which the intertemporal budget-constraint is given by

$$dA_{s,t} = rA_{s,t} + \alpha^A_{s,t} dX_t/X_t + \psi_{s,t} h_{s,t} w dt \qquad \text{if } s \leq t < s+40 \tag{3.2a}$$
$$dA_{s,t} = rA_{s,t} + \alpha^A_{s,t} dX_t/X_t - b_{s,t} dt \qquad \text{if } s+40 \leq t \leq s+60 \tag{3.2b}$$
$$A_{s,s} = 0 \tag{3.2c}$$
$$A_{s,t} \geq 0 \qquad \text{for all } s \leq t \leq s+60 \tag{3.2d}$$

for all cohorts s. In equation (3.2a), the variable $\psi_{s,t}$ denotes the accrual rate and is defined as the value of pension entitlements accrued per unit of labor supply at time t by cohort s.[3] The variable $\alpha^A_{s,t}$ represents the *direct* exposure to stock-market risk: the gains and losses from this exposure are levied immediately upon the participant via an adjustment in the value of previously accumulated pension rights $A_{s,t}$.[4] Direct adjustments in accumulated pension rights are attractive because they do not distort labor-supply choices, as will be formally shown later in this section. Each generation starts with zero pension entitlements at time s where labor-market entrance takes place. Equation (3.2d) specifies that the value of pension accumulations cannot become negative, which reflects the commonplace observation that participants in pension funds enjoy limited liability.

In addition, the pension fund keeps track of a "shadow-account" $S_{s,t}$, which represents the *financial surplus*. The introduction of the surplus-account $S_{s,t}$ is motivated by the observation that the value of pension fund assets generally deviates from the value of the liabilities (i.e. the value of accumulated pension entitlements of all participants together). Deviations between assets and liabilities result in a funding deficit or a funding surplus. Essentially, the deficit or surplus of a pension fund contains the financial gains and losses from risk-taking in the past that have not yet been recouped upon the participants of a pension fund. The pension fund can use the financial surplus in situations where it is not possible to levy financial shocks directly upon participants, i.e. in situations in which the limited-liability condition in equation (3.2d) is binding. This condition is binding for young and

[3] The variable $\psi_{s,t}$ represents both the market-value and the utility-value of pension accruals which, in contrast to chapter 2, coincide with each other.

[4] In the context of a real-life pension fund, a downward adjustment in accumulated pension rights is often referred to as a "cut" in pension rights. An upward adjustment in accumulated pension rights is often referred to as "indexation" or "inflation corrections": accumulated pension rights are compensated for (i.e indexed to) increases in price or wage inflation levels.

unborn participants who have accumulated little or no pension rights and whose wealth consists (primarily) of future labor earnings. Hence, the financial surplus $S_{s,t}$ contains the financial gains and losses from risk-taking on behalf of cohort s that will be recouped upon this generation in the future. The intertemporal budget-constraint for the surplus account $S_{s,t}$ is given by

$$dS_{s,t} = rS_{s,t} + \alpha^S_{s,t} dX_t/X_t + (\pi_{s,t} - \psi_{s,t}) w h_{s,t} dt \quad \text{if } t_0 \leq t < s + 40 \tag{3.3a}$$

$$S_{s,t_0} = 0 \tag{3.3b}$$

$$S_{s,s+40} = 0 \tag{3.3c}$$

for all cohorts s. In equation (3.3a), the variable $\alpha^S_{s,t}$ represents the *indirect* exposure to stock market risk: the gains and losses from this exposure do not directly affect the value of pension entitlements of the participant. Instead, these shocks are temporarily "stored" in the surplus account $S_{s,t}$ and are recouped upon the cohort at a later point in the working period in the form of net taxes and subsidies on labor earnings. If the contribution rate $\pi_{s,t}$ exceeds the accrual rate $\psi_{s,t}$, the pension fund levies a net tax on labor earnings, causing a funding deficit to shrink. In the opposite case, where the contribution rate falls short of the accrual rate, a net subsidy is provided on labor earnings is provided, causing a funding surplus to shrink. Equation (3.3c) imposes that a financial deficit or surplus needs to be recouped upon a participant before retirement, i.e. before human wealth has been fully depleted.

The surplus account allows the pension fund to use future labor earnings as collateral when investing in the stock market. Future labor earnings can thus be used by the pension fund to take risk on behalf of young and unborn generations, who have not yet accumulated tangible assets which give the social planner information on the distribution of earnings capacities within cohorts. Therefore, it is imposed in equation (3.3a) that the pension fund must rely on taxes and subsidies on future labor earnings to recoup the surplus $S_{s,t}$ on young and unborn generations. The power to tax or subsidize future earnings enables a pension fund to make commitments on behalf of young and unborn generations. Thereby, a pension fund is able to trade on behalf of unborn generations, and is able to use the human wealth of young investors as collateral when investing in the stock market.

Labor-supply choices are fully determined by the effective marginal wage rate, which equals the wage rate w offered by the employer plus the net value $w(\psi_t - \pi_t)$

of participation in the pension fund:

$$h_{s,t} = (w - w\pi_{s,t} + w\psi_{s,t})^{\epsilon} = h^*(1 - \pi_{s,t} + \psi_{s,t})^{\epsilon}, \tag{3.4}$$

where h^* is defined in equation (2.7). It is inferred from equation (3.4) that taxes and subsidies are proportional to labor earnings in the model, consistent with the model in chapter 2 (see equation (2.19)).

Equation (3.3b) specifies that at the initial time t_0, the time at which the pension fund is initiated, the surplus of all cohorts is initiated at zero. Cohorts that enter the labor-market after time t_0 can thus not only trade in stocks while being economically active in the period $[s,s+60]$, but also before the date of labor-market entrance in the period $[t_0,s]$. As explained in chapter 1, pre-labor-market-entry investments in the stock market enables future generations to share in current risk, and allows for an evaluation of the gains from risk sharing. The duration of the pre-labor-market-entry investment period varies across cohorts: for cohort s the duration equals $s - t_0$, where t_0 is the date at which the pension fund is initiated. Later-born cohorts thus have a longer pre-labor-market-entry investment period than earlier-born ones. Thereby, the model is in line with the approach of Ball and Mankiw (2007). In the model of Teulings and De Vries (2006), in contrast, there is no initial date t_0 specified. The specification of a time-of-initiation is convenient in this chapter, because it allows for a comparison to the model of chapter 2, which also features a date of initiation. A date of initiation is required in an evaluation of risk sharing, because otherwise the gains from risk sharing becomes infinitely large.

Pre-labor-market-entry investments result in a non-zero surplus at the age at which labor-market-entry takes place. This feature of the model resembles the common-place observation that new participants enter a pension fund while it is over- or underfunded.

It follows from equations (3.3a) and (3.3c) that the financial surplus is equal to the market value of net taxes or net subsidies on future labor earnings:

$$S_{s,t} = \begin{cases} \mathbf{E}_t \left[\int_{\max\{s,t\}}^{s+40} \frac{M_v}{M_t} w h_{s,v} \left(\psi_{s,v} - \pi_{s,v}\right) dv \right] & \text{if } t_0 \leq t < s+40 \\ 0 & \text{if } s+40 \leq t \leq s+60 \end{cases} \tag{3.5}$$

where $M.$ represents the stochastic discount factor and has been defined in equation (2.4). From equations (3.3b) and (3.5) it follows that, for all cohorts, the market value of participating in the pension fund is equal to zero from an ex-ante perspective:

$$\mathbf{E}_{t_0}\left[\int_s^{s+n}\frac{M_u}{M_{t_0}}wh_{s,u}\pi_{s,u}du\right]=\mathbf{E}_{t_0}\left[\int_s^{s+n}\frac{M_u}{M_{t_0}}wh_{s,u}\psi_{s,t}dt\right], \tag{3.6}$$

where the left-hand-side represents the market-value of pension contributions pledged to the fund while working and where the right-hand-side represents the market-value of pension accruals received in return.

Optimization problem

The pension fund maximizes the expected utility of each cohort separately. For each cohort s the pension fund maximizes preferences as specified in equation (2.5) with respect to the five policy parameters $\pi_{s,t}$, $\psi_{s,t}$, $b_{s,t}$, $\alpha^A_{s,t}$ and $\alpha^S_{s,t}$, subject to the budget constraints in equations (3.2) and (3.3) and labor-supply choices in equation (3.4). Since the optimization problem is solved for each cohort separately, the resulting policy rules are age-specific.

The model does not adopt an analytical solution and is solved numerically by using standard finite-difference methods. In particular, I make use of backward-induction, state-space discretization, spline interpolation and Gaussian quadratures.[5] The numerical solution-technique is described in Appendix 3.A.

3.3 Absence of pre-labor-market-entry investments

This section provides the solution in absence of pre-labor-market-entry participation in the stock market. Pre-labor-market-entry investments in stocks are ruled out by requiring the financial surplus to be zero at the date of labor-market entry. That is, the constraint in equation (3.3b) is replaced by:

$$S_{s,s}=0. \tag{3.3b'}$$

The structure of this section is as follows. Section 3.3.1 solves the special case where labor supply is infinitely-elastic. Section 3.3.2 treats the special case where labor supply is inelastic. Finally, section 3.3.3 treats the general solution.

[5] For an extensive treatment of finite-difference methods, see e.g. Ames (1977), Judd (1998) or Candler (1999).

3.3.1 Infinitely elastic labor supply (i.e. borrowing constraint)

If labor supply is infinitely-elastic (i.e. $\epsilon \to \infty$), it is not possible for the pension fund to levy shocks upon the participant in the form of taxes and subsidies on labor earnings. Any distortion to the marginal wage rate would result in an infinite adjustment in labor supply. Gains and losses from risk taking need to be levied directly upon the participant via adjustments in the value of pension accumulations. The pension fund is thus unable to use the human wealth of young and future generations as collateral when investing in the stock market. As a result, the optimization problem reduces into the model of Merton (1969) and Samuelson (1969) *with* a borrowing constraint.

The Merton (1969) model under the borrowing constraint has been treated in section 2.3, in which it was shown that a borrowing constraint prevents the use of future labor earnings as collateral for investments in the stock market. As a result, young and unborn generations are unable to take full advantage of the equity premium in financial markets.

3.3.2 Inelastic labor-supply (i.e. no borrowing constraint)

Under inelastic labor-supply (i.e. $\epsilon = 0$), the pension fund is able to replicate the *unconstrained* (i.e. without borrowing constraint) solution of Samuelson (1969) and Merton (1969), which has been treated in section 2.3. This section explains how this is accomplished.

The optimal consumption strategy $\{\pi_{s,t},\ b_{s,t}\}$ follows from equation (2.10), which teaches that a wealth shock at any time t is levied proportionally equally upon all remaining consumption levels:

$$\partial C_{s,u}/C_{s,u} = \partial W_{s,t}/W_{s,t}, \tag{3.7}$$

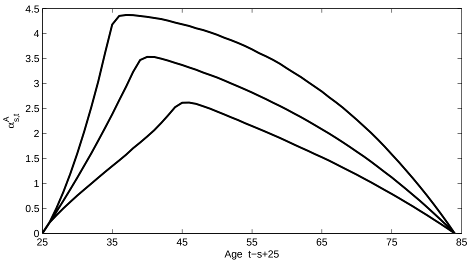
for all $u > t$, where total wealth $W_{s,t}$ is now given by the sum of pension accumulations $A_{s,t}$ and human wealth $\hat{H}_{s,t}$. In this chapter, the human wealth $\hat{H}_t \equiv H_t + S_t$ of a pension fund participant is redefined as the value of discounted future labor-earnings (i.e. $H_{s,t} = \mathbf{E}\left[\int_t^{s+40} \frac{M_v}{M_t} w h_{s,t} dv\right]$ if $t-s < 40$ and zero otherwise) corrected for the discounted value $S_{s,t}$ of taxes and subsidies on future labor-earnings. The optimal rules for the contribution rates $\pi_{s,t}$ and the benefit levels $b_{s,t}$ are obtained from substitution of equation (3.1) into equation (3.7).

There is only *one* way in which the pension fund can adjust the consumption level of workers, namely via changes in the contribution rate. There are, however, *two* ways in which the pension fund can adjust retirement consumption levels: the pension fund can adjust the value of *previously accumulated* pension rights $A_{s,t}$, or it can adjust the value of *future* pension accruals $\alpha_{s,t}$. If the pension fund adjusts previously accumulated pension rights, participants are exposed to risk in a *direct* way, via an exposure $\alpha^A_{s,t}$. In contrast, if the pension fund adjusts future pension accruals, participants are exposed to risk in an *indirect* way, via an exposure $\alpha^S_{s,t}$.

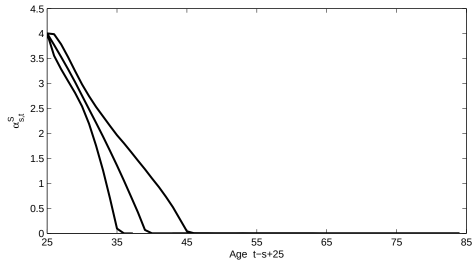
Under inelastic labor-supply, it is irrelevant in which way retirement consumption is adjusted by the pension fund. That is: it is irrelevant whether participants are exposed to financial market risk in a direct way or in an indirect way. As a result, the optimal investment strategy $\{\alpha^A_{s,t},\ \alpha^S_{s,t}\}$ is not uniquely defined. The optimal-investment strategy, however, does adopt a unique solution in terms of the *total* risk exposure $\alpha^A_{s,t} + \alpha^S_{s,t} \equiv \alpha_{s,t}$. Equation (2.11) teaches that the optimal amount invested in stocks at age t is given by a fixed fraction $\lambda/(\gamma\sigma)$ of the total wealth of the cohort:

$$\alpha_{s,t} = \frac{\lambda}{\gamma\sigma} W_{s,t}. \tag{3.8}$$

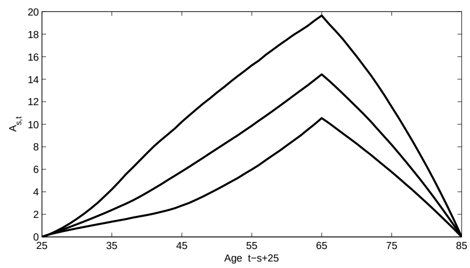
Figure 3.1 illustrates the optimal solution strategy. As previously explained, there exists no unique solution for the investment strategy $\{X^A_{s,t},\ X^S_{s,t}\}$. Therefore, Figure 3.1 shows the unique solution in which the elasticity of labor supply ϵ is positive but very small: $\epsilon = 0.001$. This case can be interpreted as the situation in which labor supply is inelastic, but the pension-fund policy does not induce unnecessary distortions in the labor market anyway. During the beginning of the life-cycle, a participant has not acquired sufficient tangible assets (i.e. pension entitlements $A_{s,t}$) for a direct exposure to equity risk. The pension fund therefore levies shocks upon young participants in the form of taxes and subsidies on future labor earnings by employing the surplus-account $S_{s,t}$. In effect, the pension fund uses future labor earnings as collateral when investing in the stock market on behalf of young workers, thereby alleviating the borrowing constraint. As the participant accumulates more pension entitlements during the working life, it becomes possible to recoup financial shocks via direct adjustments in pension entitlements.



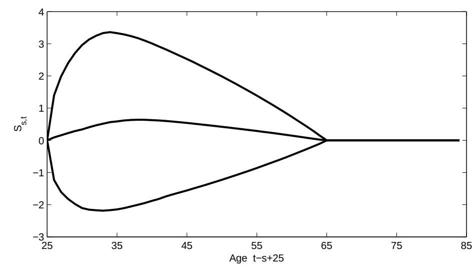
(a) Direct equity exposure $\alpha^A_{s,t}$



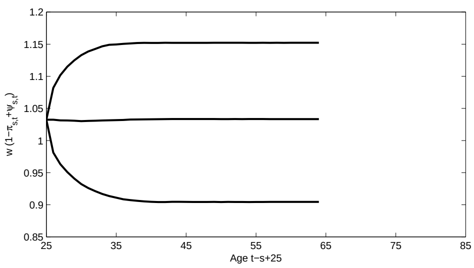
(b) Indirect equity exposure $\alpha^S_{s,t}$



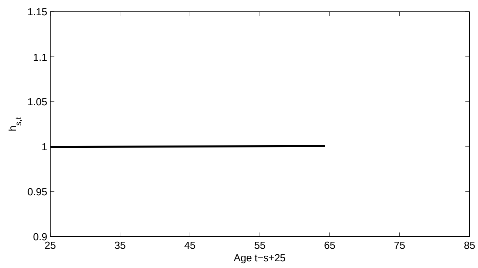
(c) Pension entitlements A_t



(d) Financial surplus $S_{s,t}$



(e) Effective wage rate $w(1-\pi_{s,t}+\psi_{s,t})$



(f) Labor supply $h_{s,t}$

Figure 3.1: *The 5%, 50% and 95% percentiles for various variables in the case where the elasticity of labor supply is very small but positive (*$\epsilon = 0.001$*). The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85.*

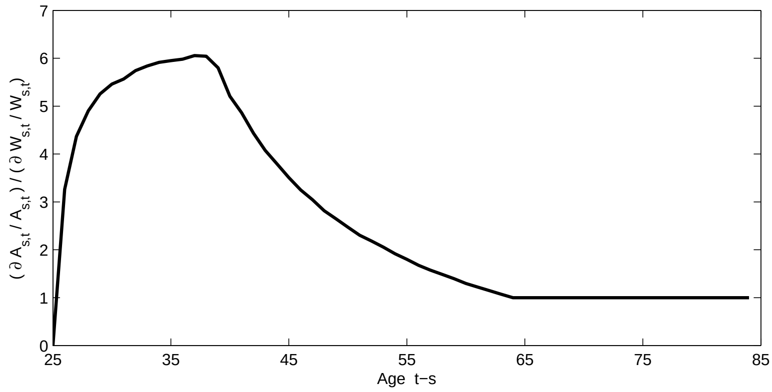
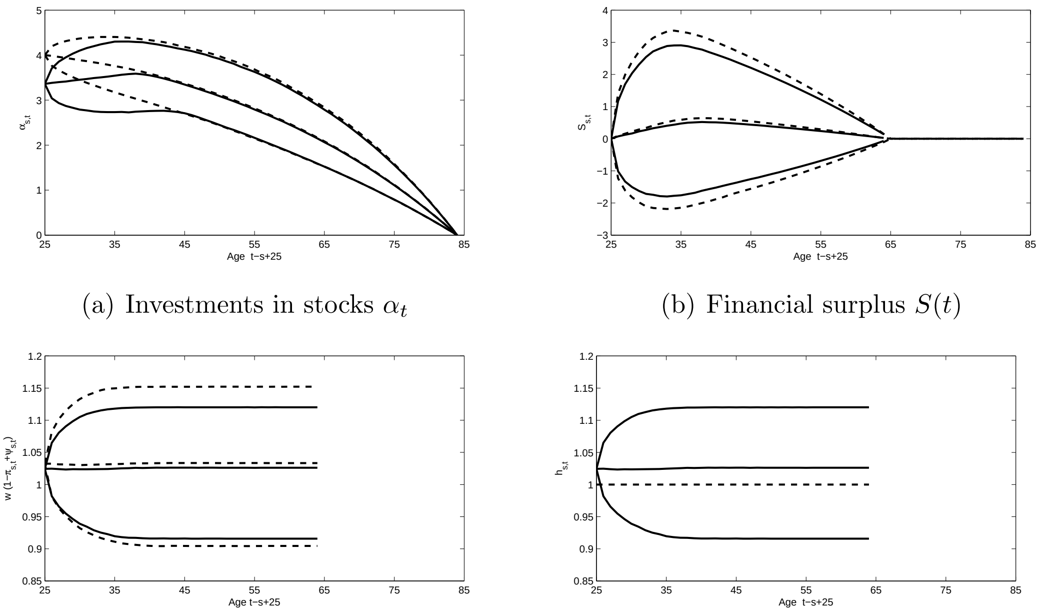


Figure 3.2: *The elasticity of the value of pension rights with respect to wealth shocks, i.e.* $(\partial A_{s,t}/A_{s,t})/(\partial W_{s,t}/W_{s,t})$*, as a function of age. Calculations are based upon the default parameters as contained in Table 2.1. The elasticity of labor supply is assumed very small but positive (*$\epsilon = 0.001$*). The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85.*

Recall that in the solution of Figure 3.1, the pension fund mitigates distortions in the labor market because the elasticity of labor supply is positive: $\epsilon = 0.001$. It is shown in Figure 3.2 that a age-specific policy mitigates distortions in the labor market via age-differentiation in relative adjustments in the value of pension rights. In the optimal pension fund policy, the value of pension rights of young workers is approximately six times *more* elastic to economic shocks than the pension rights of old workers and retirees. Hence, the optimal solution features age-differentiation in relative adjustments in the value of pension rights. The intuition for this result is that the wealth of young workers primarily consists of future labor earnings. As explained in the introduction of chapter 2, lump-sum transfers cannot be applied when future labor earnings are used as collateral. Hence, to mitigate distortions in the labor market, the pension fund finds it is optimal to use previously accumulated pension rights as collateral whenever possible. Given that the amount of pension rights accumulated by young workers is small, the value of pension rights of young workers becomes very responsive to economic shocks in the optimal solution.



(a) Investments in stocks α_t

(b) Financial surplus $S(t)$

(c) Effective wage rate $1 - \pi_t + \psi_t$

(d) Labor supply h_t

Figure 3.3: *The 5%, 50% and 95% percentiles for a number of variables for the case where labor supply is elastic with* $\epsilon = 1$ *(solid lines). Also shown is the case where labor supply is very small but positive with* $\epsilon = 0.001$ *(dashed lines). Calculations correspond to the benchmark parameters as contained in Table 2.1. The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85.*

3.3.3 Elastic labor supply (i.e. endogenous borrowing constraint)

Figure 3.3 shows the optimal solution strategy under endogenous labor supply. Labor-market distortions reduce the attractiveness of using the human wealth of the participant as a collateral when investing in the stock market. This leads to a reduction in the risk-bearing capacity of young workers in comparison to the case in which labor supply is inelastic. Under elastic labor supply, it becomes less attractive to use future labor earnings as collateral when investing in the stock market on behalf of young generations. Elastic labor supply therefore, essentially, introduces an *endogenous* borrowing constraint: borrowing against human capital is still possible, but it becomes costly. Figure 3.3 suggests that the welfare costs from distortions in the labor market are relatively mild in the optimal solution, because the reduction in stock investments on behalf of young workers is modest.

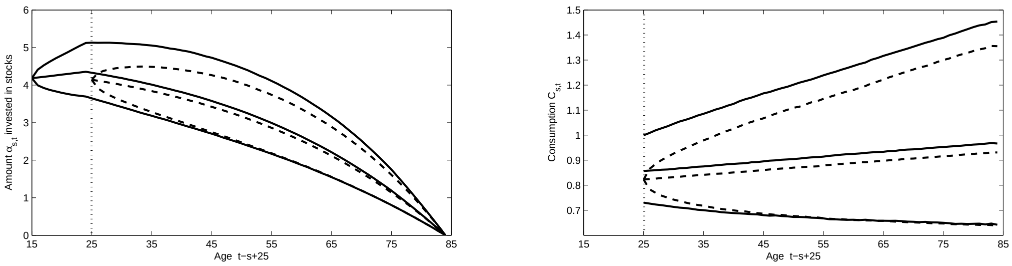
3.4 Pre-labor-market-entry stock-market participation

This section introduces pre-labor-market-entry investments in stocks. This is accomplished by using the constraint in equation (3.3b') instead of the one in equation (3.3b). Section 3.4.1 discusses the solution under inelastic labor supply. Section 3.4.2 treats the case where labor supply is elastic.

3.4.1 Inelastic labor supply

Pre-labor-market entry stock-market participation under inelastic labor supply has been examined in Teulings and De Vries (2006). Their solution is briefly discussed in this section for the sake of completeness. The optimal solution strategy after the date of labor-market-entrance remains unchanged. Before the date of labor-market-entry, there is no consumption decision, so we only have to solve for the optimal investment strategy. Teulings and De Vries (2006) show that the investment strategy before labor-market-entry is given by equation (3.8), and is thus essentially the same as the investment strategy after labor-market-entry. Notice, however, that there are no pension accumulations before the date of labor-market-entrance, implying that the investment strategy is based upon $W_{s,t} = \hat{H}_{s,t}$, i.e. the discounted value $H_{s,t}$ of future labor-earnings (i.e. $H_{s,t} = \mathbf{E}\left[\int_s^{s+40} \frac{M_v}{M_t} w h_{s,t} dv\right]$) corrected for the market value $S_{s,t}$ of taxes and subsidies on future labor-earnings.

Figure 3.4 illustrates the solution for the benchmark parameters. The Figure takes the perspective of the cohort that enters the labor market at time $t_0 + 10$, 10 years after the initiation of the fund. Figure 3.4 shows that the demand for stocks before the date of labor-market entrance is large. Due to the increase in risk taking, consumption becomes higher on average (due to the risk premium), but also more volatile.



(a) Amount $\alpha_{s,t}$ invested in stocks (b) Consumption $C_{s,t}$

Figure 3.4: *The 5%, 50% and 95% percentiles for a number of variables for the case with (solid lines) and without (dotted lines) pre-labor-market entry investments in the stock market. The duration of the pre-labor-market-entry investment period equals 10 years. That is: we are considering the cohort that enters at time t_0+10, 10 years after initiation of the fund. The figure is based upon the benchmark parameters as contained in Table 2.1. The parameter of labor supply elasticity is set equal to $\epsilon = 0$. The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85. The dotted vertical line at age 25 represents the date of labor-market-entry. The pension fund invests in the stock market on behalf of the participant starting from the age of 15.*

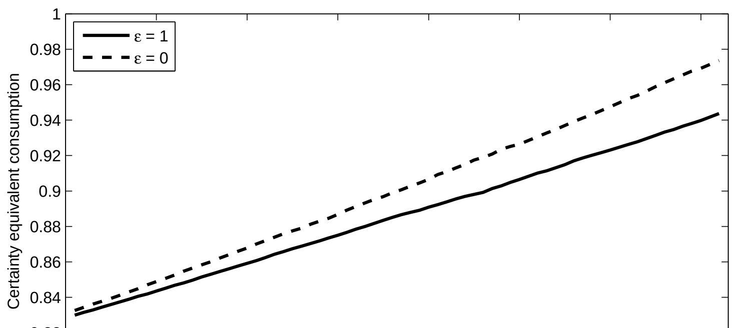


Figure 3.5: *The welfare gain from pre-labor-market-entry stock-market participation as a function of its duration. Calculations correspond to the benchmark parameters in Table 2.1.*

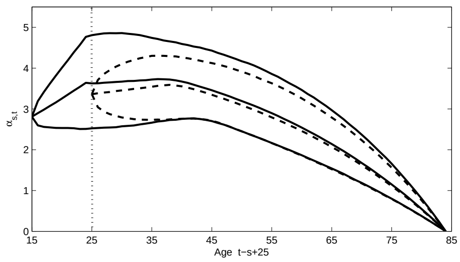
Figure 3.5 illustrates the certainty equivalent consumption level as a function of the duration of the pre-labor-market-entry investment period. Pre-labor-market entry investments in the stock market substantially increase ex-ante welfare levels, because it allows participants to take more advantage of the equity premium in financial markets. As a result, labor-born cohorts, with a long pre-labor-market-entry investment-period, enjoy a higher welfare level than earlier-born ones. Hence, by treating generations *equal* in market-value (see equation (3.6)), the pension fund treats generation *unequal* in welfare terms: later-born cohorts profit more from risk-sharing than earlier-born ones. The differences are large. For example, under inelastic labor supply, the cohort that enters the pension fund 40 years after the time of initialization, at time $t_0 + 40$, is (0.9105-0.8296)/0.8296=9.8% better off than the cohort that enters 40 years earlier, at time t_0.

3.4.2 Elastic labor supply

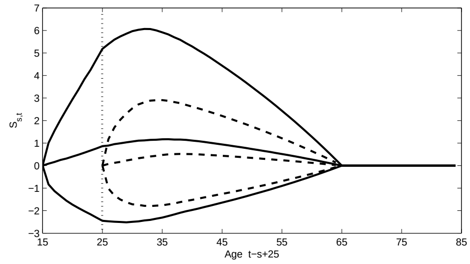
Figure 3.6 illustrates the solution under elastic labor supply. Pre-labor-market entry investments in the stock market result in a non-zero financial surplus at the date of labor-market entrance. This causes wage-differentials to become more uniformly distributed over the working-life, because participants enter in a pension fund with a funding surplus or shortfall.

3.5 Intergenerational fairness

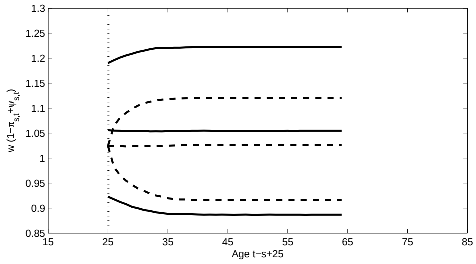
In two ways, the model in chapter 3 differs from the model in chapter 2. First, the contribution and benefit rules of the pension fund were imposed *uniform* in chapter 2, whereas section 3.2 uses a *age-specific* policy. Second, the two models differ in terms of their *criterion for intergenerational fairness*. Generations are treated equal in *market terms* in chapter 3, as specified in equation (3.6). In contrast, generations are treated equal in *welfare terms* in chapter 2, as specified in equation (2.24). Figure 3.5 has pointed out that these two fairness criteria are not the same. Therefore, this section imposes the fairness criterion of chapter 2 upon the model of chapter 3. Thus, consistent with equation (2.24), it is imposed



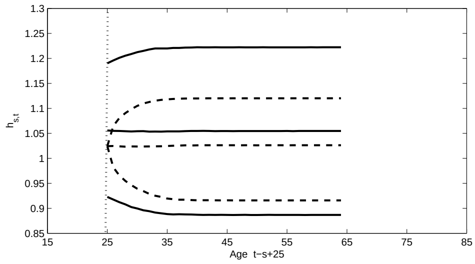
(a) Equity exposure $\alpha_t^A(t) + \alpha_t^S$



(b) Surplus S_t



(c) Effective wage rate $1 - \pi_t + \alpha_t$



(d) Labor supply h_t

Figure 3.6: *The 5%, 50% and 95% percentiles for a number of variables for the case with (solid lines) and without (dotted lines) pre-labor-market entry investments in the stock market. The duration of the pre-labor-market-entry investment period equals 10 years. That is: we are considering the cohort that enters at time* $t_0 + 10$, *10 years after initiation of the fund. The figure is based upon the benchmark parameter values. The parameter of labor supply elasticity is set equal to* $\epsilon = 1$. *The modeling outcomes are expressed in terms of age, where it is assumed that the individual investor enters the labor force at age 25, works until age 25+40=65 and is subsequently retired until age 65+20=85. The dotted vertical line at age 25 represents the date of labor-market-entry.*

that:

$$U_s = U_{s'} \qquad \forall s, s' > t_0 \tag{3.9}$$

where U_s is specified in equation (2.5). The restriction in equation (3.9) accomplishes that the *only* remaining difference in comparison with the model of chapter 2 is the nature of the policy rules (uniform vs age-specific), allowing me to calculate the welfare gain associated with age-specific policy rules.

To satisfy the constraint in equation (3.9), it is required that ex-ante redistributive transfers (in market value) between cohorts are introduced to the model. Ex-ante redistributive transfers can be implemented in many different ways. It turns out to be convenient to choose the approach where all redistribution takes place at the initial time t_0 where the pension fund is initiated. That is: generations can start with a positive or negative surplus S_{s,t_0} at time t_0, thereby relaxing the restriction in equation (3.3b). This is a very convenient approach to introduce ex-ante redistribution to the model, because it follows straightforwardly from equation (3.6) that the market value of participating in the pension fund for any cohort s is negative (positive) if its initial surplus S_{s,t_0} is negative (positive). Hence, the ex-ante redistribution between cohorts follows directly from the differences in the value of the initial surplus S_{s,t_0}.

Notice that equation (3.9) specifies the welfare criterion only for cohorts that are born after the date t_0, i.e. the cohorts $s \in \mathbb{N} : s \geq t_0$. The transition cohorts, i.e. the cohorts $s \in \mathbb{N} : t_0 - n - m < s < t_0$, are provided with pension entitlements A_{s,t_0} at a level that ensures that their gain from joining the pension fund is the same as their gain in the social planner approach of chapter 2. At the date of the transition, the pension fund satisfies the budget constraint:

$$F_{t_0} = \sum_{s: s \in \mathbb{N}: s \geq t_0} S_{s,t_0} + \sum_{s: s \in \mathbb{N}: t_0 - 60 < s < t_0} A_{s,t_0}, \tag{3.10}$$

where $F_{t_0} = 388wh^*$ is the size of the wealth transfer at the transition date t_0 and has been specified in chapter 2.

Figure 3.7 shows the ex-ante redistribution in the optimal solution for the benchmark parameters. Ex-ante redistribution is expressed in terms of the initial value of the surplus S_{s,t_0}. A negative value of the initial surplus implies that that a cohort loses from the risk sharing contract in ex-ante market terms. A negative value, in contrast, implies that a cohort gains. The Figure shows that the market

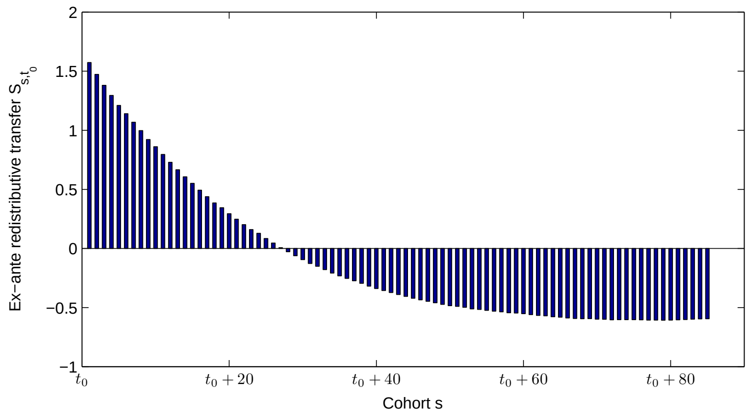


Figure 3.7: *The ex-ante redistribution in market terms between generations if all generations are treated equal in welfare terms, as specified in equation* (3.9)*. Calculations correspond to the benchmark parameters contained in Table 2.1. The elasticity of labor supply is set equal to zero.*

value of participation is positive for earlier-born cohorts (the cohorts that enter the pension fund before date $t_0 + 27$), while later born-cohorts are worse off in market terms. Hence, it follows that treating cohorts *equal* in welfare-terms (as specified in equation (3.9)) implies that cohorts are treated *unequal* in terms market-value. The size of the redistribution in market terms between cohorts is substantial. For example, for the cohort that enters the pension fund at time t_0, the market value of participation is equal to a one-time bonus of 1.57 times annual earnings received at the beginning of the working period. Given that the pension fund is a zero-sum game in terms of market value, the "gifts" to earlier-born cohorts need to be financed by later-born generations. Hence, later-born cohorts lose in terms of market value.

Figure 3.8 shows the welfare effects of the pension fund in chapter 3 under the constraint for intergenerational fairness in equation (3.9), in which welfare is expressed in terms of certainty equivalent consumption levels. Since all cohorts have the same ex-ante welfare level, the welfare levels in Figure 3.8 apply to all cohorts. Figure 3.8 is the same as Figure 2.5, except that the policy of the pension fund is differentiated with respect to age in the model of chapter 3, whereas chapter 2 applied uniform policy rules. Comparing Figures 2.5 and 3.8, it follows that

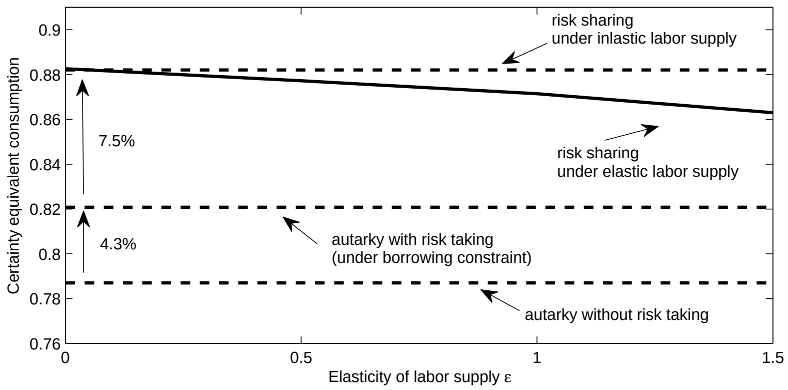


Figure 3.8: *The certainty-equivalent consumption level in the pension fund as a function of the wage-elasticity of labor supply ϵ. The welfare level applies to all cohorts, since all cohorts enjoy the same welfare level due to the restriction in equation* (3.9). *Calculations are based upon the default parameters as contained in Table 2.1.*

distortions in the labor market are dramatically amplified by uniform policy rules in comparison to age-specific rules. Labor-market distortions are greatly reduced if age-specific policy rules are applied. For the benchmark parameters, I find that labor market distortions erode only 18.0% of the welfare gain from risk sharing under age-specific policy rules if the elasticity of labor supply is equal to 1, whereas this is as much as 80.6% under uniform policy rules (derived in chapter 2).

Finally, there is another important difference in the welfare effects between chapters 2 and 3. Under the age-specific policy rules of chapter 3, the risk sharing solution is always Pareto-efficient. After all, if the elasticity of labor supply goes to infinity, welfare levels in the pension fund converge to the welfare level in autarky, as shown in section 3.3.1. In contrast, a Pareto-efficient risk sharing solution does not always exists when uniform policy rules are used. This was illustrated in Figure 2.5, where it was found that risk sharing is not Pareto-efficient anymore if the elasticity of labor supply exceeds the value of 1.1.

3.6 Conclusions

This chapter shows that age-specific policies can be very effective in mitigating distortions in the labor market. Distortions in the labor market are reduced if relative adjustments in the value of accumulated pension rights are differentiated with respect to age. In the optimal policy, the value of pension rights of young workers are responsive to economic shocks than those of old workers. Thereby, the result in this chapter provide a new argument in favor of age-differentiation in collective pension funds, as promoted in Molenaar, Munsters, and Ponds (2008).

3.A Appendix

Description of Numerical Method

The solution for the borrowing constrained individual as well as the solution for the pension fund participant are derived through numerical solution techniques. This section explains the solution technique for the more complex problem of the pension fund participant. The procedure for the borrowing constrained individual is less complicated and is solved in a similar way.

During the working period the problem contains two state variables (A_t and S_t) while during the retirement period, in which the financial surplus is zero, A_t is the only state variable. I assume that the labor supply decision h_t of the pension fund participant is given by the approximation rule given in equation (3.4), so that there are four decision variables to be solved during the working period (π_t, ψ_t, α_t^A and α_t^S) and two decision variables during the retirement period (b_t and α_t^A). I apply a discretization with respect to age with time step $\Delta t = 1$ so that the time grid is given by $25-B$, $25-B+1$, ... 85. In addition, I apply discretization in both dimensions of the state space. I apply an exponentially spaced in A_t-dimension, resulting in relatively many gridpoints at low values of A_t. The state space is bounded from below in the A_t-dimension as a result of the constraint in equation (3.2d). The decision rules can now be represented on a numerical grid in the state space at all ages t on the age grid.

The decision rules are solved via backward induction. For every age age t on the grid prior to age T, and for each point in the state space, I optimize utility at time t with respect to the decision variables using a grid search. Discretization of the utility function with respect to age implies that utility $U(t)$ at age t is approximated by $U(t) = \Delta t\, u(C(t), h(t)) + e^{-\beta \Delta t} \mathbf{E}_t \left[U(t + \Delta t)\right]$, where the first term on the right-hand-side represents the utility gained at present and where the second term represents the discounted expected continuation value. The utility gained at present follows directly from the decision variables. The expected continuation value can be computed once the problem at time $t + \Delta t$ is solved. If continuation values do not lie on the state space grid, the value function is evaluated on the basis of spline interpolation. In the final period, the solution is trivial due to the absence of a bequest motive: the individual finds it optimal to consume all remaining wealth. This provides the terminal condition required for the backward induction procedure. The numerical integrations are calculated by using Gaussian quadrature, allowing for an accurate approximate of the continuous distribution of asset returns.

The 5%, 50% and 95% percentiles reported in the paper are generated by Monte-Carlo simulations on the basis of the numerically solved decision variables. If the simulated values for the state variables do not lie on the state space grid, decision making is evaluated on the basis of cubic interpolation of decision rules represented on the state space grid.

Chapter 4

Risk Sharing and Long-Run Labor Income Risk

This chapter evaluates how labor-income risk affects the gains from risk sharing. Labor-income risk can make it less attractive for unborn generations to share in current financial risk. Comovements between stock and labor markets cause the human wealth of unborn generations to become correlated with current financial shocks. This reduces the risk appetite of future generations and reduces the attractiveness of risk sharing. The stylized analysis of section 1.2.5 is extended to a continuous-time overlapping generations model in which stock and labor markets are cointegrated. The model abstracts from elastic labor-supply, which has been treated in chapters 2 and 3.

4.1 Introduction

Risk sharing between non-overlapping generations in pension funds can be welfare improving. Financial shocks are smoothed over many generations so that each generation bears only a small portion of the risk. The economic intuition behind the welfare gain from risk sharing is that there is an improvement in the time-diversification of risk: risks are spread over a broader base (i.e. a larger number of generations) than is possible in financial markets in which only overlapping generations are able to trade risk with each other.[1] If financial shocks are smoothed over a larger number of generations, then each generation bears only a small portion of the risk. This increases the risk-bearing capacity of society because no single generation is confronted with a large shock.

Many studies on risk-sharing in pension funds abstract from labor-income risk, e.g. Teulings and De Vries (2006), Gollier (2008) and Cui, De Jong, and Ponds (2011). These studies find that it is optimal for a pension fund to spread financial-market risk over a large number of generations. By smoothing financial shocks over many generations, the time-diversification of risk is improved and welfare is increased. Most existing risk-sharing studies therefore suggest that currently-living generations should bear only a fraction of current financial shocks, while the rest is smoothed over as many future generations as possible. Risk is shifted far into the future and the optimal recovery period (i.e. the period during which the pension fund recovers from financial shocks) is long. In these studies, a short recovery period is suboptimal. A short recovery period implies that generations that are born far into the future are unaffected by current financial shocks, so that risk is not optimally smoothed across generations.

This chapter points out that labor-income risk crucially determines the optimal risk-sharing rules. In particular, it is shown that long recovery periods are no longer optimal once labor-income risk is recognized. It is no longer optimal to shift risk far into the future. The numerical results in this chapter indicate that the optimal recovery period of a pension fund is relatively short. In the optimal solution, currently-living generations absorb the majority of current financial shocks by themselves, instead of shifting risk into the future.

The optimal recovery period can be expressed in terms of the half-life of the recovery process, i.e. the half-life of the impact of a financial shock on the financial position of the pension fund. In the absence of labor-income risk, the half-life of the recovery process is *infinite*: financial shocks are smoothed over all future generations and therefore have a permanent impact on the financial position of the pension fund. In the presence of labor-income risk, I find that the half-life of the optimal recovery process becomes *finite*. Depending on the parameterization

[1] This point was made by Diamond (1977), Merton (1983) and Gordon and Varian (1988). More recent contributions include Smetters (2006), Bohn (2006), Ball and Mankiw (2007), Gollier (2008), Gottardi and Kubler (2008) and Cui, De Jong, and Ponds (2011).

of the model, the optimal half-life is somewhere between 5 and 19 years. The intuition for this result is that it becomes less attractive for future generations to share in current financial risks if these risks relate positively to their future labor earnings. If stock and labor markets move together in the long run, then it becomes unattractive for future generations to share in current risks, because they are already affected by current risks via their human wealth. In the presence of labor-income risk, there is thus a limited role for a pension fund to shift current risk into the future.

The stylized framework of section 1.2.5 is extended to a rich stochastic modeling environment for stock and labor markets. The model has a common risk factor for stock and labor markets and features the property that stock and labor markets move together at long horizons, mirroring changes in the broader economy. The economic intuition behind this assumption is that a sustained period of high (low) economic growth results in strong (weak) performance of both stock and labor market over the long run. As a result, stock and labor markets move together in the long-run so that the factor shares of labor and capital are stationary. The long-run restriction that the factor shares of labor and capital are stationary is suggested by the form of most production functions used in macroeconomic theory. If labor and capital income are allowed to have independent trends (whether deterministic or stochastic), then the factor share of labor will approach zero asymptotically (if capital income grows faster than labor income) or the factor share of capital will approach zero (in the opposite case). This is contrary to what the data shows: although factor shares vary over time, they show no tendency to converge to zero or one.

Indeed, Benzoni, Collin-Dufresne, and Goldstein (2007) provide empirical evidence for this hypothesis, by showing that labor income and dividends on stock holdings are co-integrated. The estimates for the cointegration coefficient are significant, but fall in a wide range. They find an estimate for the cointegration coefficient of 0.205 when using US data going back to 1929, while the estimate is as low as 0.0475 when relying on the post-World War II sample period.

The economic modeling environment in this chapter is adopted from Benzoni, Collin-Dufresne, and Goldstein (2007), in which labor earnings are cointegrated with dividends on the stock portfolio. They show that cointegration causes the human capital of young investors to become strongly correlated with stock returns, which reduces their risk appetite. In contrast to other studies that ignore long-run labor income risk, they find that can even be optimal for young investors to take a short position in stocks, as this provides a hedge against future labor income shocks. This result is able to explain the high-levels of non-participation in stock markets by young investors (the stock participation puzzle).

Consistent with empirical findings, the cointegration-framework of Benzoni, Collin-Dufresne, and Goldstein (2007) allows for a low (or zero) contemporaneous correlation between labor income shocks and stock returns, whereas long-run correlations can be high.[2] Accounting for horizon-dependent correlations is important in the analysis in this chapter: different generations face different investment horizons and are thus affected in different ways by economic shocks. Furthermore, the model in this chapter is in continuous-time time and is therefore able to provide insights about the half-life of the optimal recovery period of a pension fund.

Many other recent papers have assumed that labor income and dividend flows are cointegrated, see e.g. Baxter and Jermann (1997), Menzly, Santos, and Veronesi (2004), Santos and Veronesi (2006) and Geanakoplos and Zeldes (2010). Alternative long-run specifications for the interrelation between stock returns and aggregate labor income have been examined in Storesletten, Telmer, and Yaron (2004), Storesletten, Telmer, and Yaron (2007) and Lynch and Tan (2008). Earlier studies that investigate a link between aggregate labor income and asset prices include Mayers (1974), Fama and Schwert (1977), Black (1995), Jagannathan and Kocherlakota (1996), and Campbell (1996). In the study of Campbell (1996), a high correlation between human capital and market returns is obtained in a model in which there is no strong interrelation between stock and labor markets. Campbell (1996) uses the same highly time-varying discount factor to discount both labor income and dividends. As a result, the correlation between human capital and market returns is high due to the common and highly time-varying discount

[2] Many studies feature low (or zero) correlations between aggregate labor income and stock returns, both contemporaneously as well as at long horizons. See e.g. Lucas and Zeldes (2006), Jagannathan and Kocherlakota (1996), Sundaresanz and Zapatero (1997), Carroll and Samwick (1997), Gourinchas and Parker (2002), Campbell, Cocco, Gomes, and Maenhout (2001), Cocco, Gomes, and Maenhout (2005), Davis and Willen (2000), Gomes and Michaelides (2005), Haliassos and Michaelides (2003), Viceira (2001). The assumption of low correlations at long horizons is controversial, given the empirical evidence provided in Benzoni, Collin-Dufresne, and Goldstein (2007).

factor.

This chapter contributes to the existing literature in three ways. First, it is shown that the optimal recovery period of a pension fund is crucially determined by the long-run dynamics of labor income. The numerical results in this chapter indicate that long recovery periods are no longer optimal in the presence of labor-income risk. The optimal recovery period of a pension fund is relatively short. I find that the half-life of the optimal recovery process is somewhere between 5 and 19 years, depending on the parameterization of the model. Hence, currently-living generations absorb the majority of financial shocks by themselves in the optimal risk-sharing solution, instead of shifting risk onto unborn generations.

Second, this chapter shows that the gains from risk sharing are dramatically reduced once the long-run dynamics of labor income are recognized. For the benchmark parameter values, 70% of the gain from risk sharing is eroded by long-run labor-income risk. The economic intuition for this result is that comovements between stock and labor markets cause the human wealth of future generations to become correlated with financial shocks. This reduces the risk appetite of future generations and hence reduces the attractiveness of risk sharing. Interestingly, the effect of cointegration on risk sharing is large regardless of the horizon at which comovements between stock and labor markets takes place. Even if cointegration takes place at a very long horizon (i.e. if the cointegration coefficient is small) the gain from risk sharing is reduced by more than half. The intuition for this result is that the human capital of unborn generations has a long duration and hence correlates with current stock returns, regardless of whether cointegration between stock and labor markets takes place at a horizon of 5 years, 10 years or 20 years.

As the third contribution to the literature, the model of Benzoni, Collin–Dufresne, and Goldstein (2007) is generalized to the case where stock returns are affected by risk sources other than dividend shocks. The framework of Benzoni, Collin-Dufresne, and Goldstein (2007) assumes that dividend shocks are the single source of variation in stock returns. This assumption leads to a very strong interrelation between stock and labor markets: both dividend shocks as well as labor income shocks are closely related to future productivity levels. The assumption can therefore overstate the implications of cointegration, because variation in stock returns can also be driven by factors that are less or not related to future productivity levels, such as asset bubbles, mispricing or time-variation in discount rates. To prevent the implications of cointegration from being overstated, I introduce sources of variation in stock returns other than dividend shocks. I find that this modeling extension dramatically reduces the effect of cointegration on the portfolio holdings of individual investors. In particular, I find that the negative demand for stocks by young investors, reported in Benzoni, Collin-Dufresne, and Goldstein (2007), is not robust with respect to alternative parameter choices.

Van Hemert (2005) and Bohn (2009) also examine optimal risk-sharing in a setting in which stock and labor markets are subject to a common risk factor. In the framework of Van Hemert (2005), labor income and capital returns follow a joint Markovian process, thereby allowing for horizon-dependent correlations. However, the Markov process in Van Hemert (2005) is imposed to be stationary, implying that labor income is not risky in the long run. Bohn (2009) uses a VAR model to estimate 30-year correlations between productivity and capital returns. He reports a positive correlation between 30% and 60%, depending on the specification of the VAR model. Bohn (2009) finds that, due to risky labor income, workers bear systematically more risk than retirees. Efficient risk-sharing policies should therefore shift risk away from workers to retirees. He concludes that safe pensions can be rationalized as efficient only if preferences display age-increasing risk aversion, such as habit formation.

The structure of the remainder is as follows. Section 4.2 introduces the model. Section 4.3 treats the autarky situation in which individuals save and invest on an individual retirement account. Section 4.4 derives the risk-sharing solution. Finally, section 4.5 concludes.

4.2 The model

The model assumes labor supply to be inelastic: individuals are unable to adjust their working hours and are unable to adjust their retirement age. The model thus abstracts from elastic labor-supply, which has been treated in chapters 2 and 3.

To reduce the computational complexity of the optimization problem, consumption patterns are assumed to be more simple than in chapters 2 and 3. Two simplifying assumptions are made. First, the model assumes a constant savings/contribution rate. Second, the model abstracts from investments in the stock market during retirement: wealth is converted into a flat annuity at the retirement date. Due to these two assumptions, the social planner is not required to solve for the contribution rate for each working cohort and the benefit payout level for each retired cohort at each point in time.

Notice that the assumption of a constant savings/contribution rate has not been made in chapters 2 and 3, because the assumption distorts labor-supply choices, see the explanation on page 52 in chapter 2. The current chapter abstracts from endogenous labor supply, so that this problem does not arise. The two assumptions do not affect the qualitative results in an assessment of risk sharing and are also used by Gollier (2008). Quantitatively, the two assumptions do affect results because they lead to a reduction in the risk bearing capacity of individuals, as explained op page 59 in chapter 2.

4.2.1 Overlapping generations and preferences

The framework with respect to overlapping generations is the same as in chapters 2 and 3. Thus, there are $n+m$ overlapping generations: n working generations and m retired generations. Each generation participates in the labor market for a period of n years and is subsequently retired for a period of m years. All generations are equal in size and the size of each generation is normalized to unity.

The model assumes a constant savings rate. As a result, consumption levels during the working period do not have to be included in the utility function, since they are exogenous and do not affect the optimization problem. Instead, individuals have expected utility from the flat payoff level b_{s+n} from the annuity that is purchased at the retirement date $s+n$:

$$U_s = \mathbf{E}\left[\frac{1}{1-\gamma} b_{s+n}^{1-\gamma}\right], \tag{4.1}$$

where γ represents the coefficient of constant relative risk aversion.

4.2.2 Stock and labor market

The modeling framework for stock and labor markets is an extension of Benzoni, Collin-Dufresne, and Goldstein (2007) to the case in which variation in stock returns is also driven by sources of risk other than dividend shocks.

Consider an economy consists of two assets: a riskless asset and portfolio of stocks. The riskless asset offers a real instantaneous return r. Let dividends D_t on the stock portfolio be given by Geometric Brownian Motion:

$$\frac{dD_t}{D_t} = g_d dt + \sigma_3 dz_{3,t}, \tag{4.2}$$

where $dz_{3,t}$ is a standard Wiener process and where g_d denotes the growth rate of dividends. Assuming the price of risk to be constant, and defining the stock price as the discounted value of future dividends, Benzoni, Collin-Dufresne, and Goldstein (2007) show that the excess return on the investment strategy X_t that reinvests all proceeds (dividends and capital gains) in the stock market portfolio is given by: $dX_t/X_t = \mu dt + \sigma_3 dz_{3,t}$, in which μ denotes the expected excess return on stock holdings and in which the volatility of stock returns is given by σ_3. In this specification, all the variation in stock returns is due to dividend shocks. In addition, the volatility of stock returns is equal to the volatility of dividends. As explained in section 1.2.5, these two model features are unattractive. The volatility of stock returns is observed to be substantially larger than the volatility of dividends. Stock returns are likely to be affected by risk sources other than dividend shocks, such as mispricing, asset bubbles and time-variation in discount rates. If these other risk factors are ignored, the effects of cointegration on portfolio holdings may be misguided. Therefore, let the specification for stock returns in Benzoni, Collin-Dufresne, and Goldstein (2007) be extended as follows:

$$\frac{dX_t}{X_t} = \mu dt + \sigma_2 dz_{2,t} + \sigma_3 dz_{3,t}, \tag{4.3}$$

where $dz_{2,t}$ is a standard Wiener process independent of $dz_{3,t}$ and captures sources of stock return variation other than dividend shocks. Parameter μ denotes the expected excess return on stocks and the volatility of the stock portfolio is given by $\sigma_2^2 + \sigma_3^2 \equiv \sigma^2$.

The labor income process does not include a career-pattern over the life-cycle and abstracts from individual-specific and cohort-specific shocks.[3] As a result, all working individuals earn the same labor income level L_t at each point in time t.

[3]Individual-specific shocks are ignored because I focus on *inter*generational transfers. Individuals can insure themselves against individual-specific shocks via *intra*generational risk-sharing, although this is difficult in practice due to problems related to moral-hazard, adverse-selection and limited liability. Hence, individuals will typically have to rely on self-insurance to protect against individual-specific income shocks. Individual-specific labor income shocks may be quite

Let the variable $y(t)$ denote the logarithmic labor-to capital income ratio:

$$y_t = l_t - d_t - \overline{ld}, \tag{4.4}$$

where $l_t = log[L_t]$ and $d_t \equiv log[D_t]$, and where the constant $\overline{ld}$ is the long-run logarithmic ratio of aggregate labor income to dividends. Following Benzoni, Collin–Dufresne, and Goldstein (2007), the aggregate labor labor income process L_t and the dividend process D_t are assumed to be cointegrated, by modeling the (logarithmic) labor-to capital income ratio as a mean-reverting process:

$$dy_t = -\kappa y_t + \nu_1 dz_{1,t} - \nu_3 dz_{3,t}, \tag{4.5}$$

where $z_{1,t}$ is a standard Brownian motion independent from $z_{2,t}$ and $z_{3,t}$, and where the cointegration coefficient κ measures the speed of mean reversion for the process y_t.[4] By applying Ito's lemma to equation (4.2), and substituting equations (4.4) and (4.5), it follows that the (logarithmic) income process l_t is given by:

$$dl_t = \left\{ -\kappa y_t + g_d - \frac{\sigma_3^2}{2} \right\} dt + \nu_1 dz_{1,t} + (\sigma_3 - \nu_3) dz_{3,t}. \tag{4.6}$$

Since $z_{1,t}$ is orthogonal to $z_{3,t}$, it follows that the contemporaneous correlation between stock returns and labor income shocks is given by:

$$\text{corr}(dlog[X_t], dlog[L_t]) = \frac{\sigma_3 - \nu_3}{\sqrt{v_1^2 + \sigma_2^2 + (\sigma_3 - \nu_3)^2}}. \tag{4.7}$$

Hence, the contemporaneous correlation between labor income and stock returns is governed by the term $\sigma_3 = \nu_3$. In the special case where $\sigma_3 = \nu_3$, labor income is contemporaneously uncorrelated with stock returns. At longer horizons, however, correlations are governed by cointegration whenever $\kappa > 0$. Cointegration causes the correlation between stock returns and labor earnings to be an increasing function of the horizon. Due to the presence of the risk sources $z_{1,t}$ and $z_{2,t}$, the long-run correlation is not perfect. The risk source $z_{1,t}$ represents temporary labor

relevant for our analysis: whether or not the government takes care of insuring idiosyncratic labor income risk has implications for portfolio choices and the risk sharing policies of the pension fund. For simplicity, however, individual-specific labor income risk is not included in the analysis.

[4]In the presence of cointegration, i.e. if $\kappa > 0$, the term $z_{1,t}$ captures temporary income shocks and has only a minor effect on decision making. Inclusion of the term, however, is important when calibrating the model to the data.

income shocks that do not affect the stock market. The risk source $z_{2,t}$ represents variation in stock returns that do not affect the labor market.

Benzoni and Chyruk (2009) clarify how the co-integration framework relates to the more traditional specifications for labor income risk. They show that, in absence of cointegration, i.e. if $\kappa \to 0$, and in absence of an instantaneous correlation between labor income and stock returns, i.e. if $\sigma_3 = \nu_3$, the specification is nearly identical to a framework with time-invariant correlations as in Cocco, Gomes, and Maenhout (2005). In this situation, stock returns and labor earnings follow independent random walks, and the term $z_{1,t}$ captures permanent income shocks.[5] If, in addition to these conditions, ν_1 is set equal to zero, the model features deterministic labor earnings, which grow at a rate equal to g_D, i.e. $L_v = L_t e^{g_D(v-t)}$ for all $v > t$. Labor earnings become constant, as in chapters 2 and 3, by additionally setting $g_D = 0$. Notice, however, that the models in chapters 2 and 3 do not assume a constant contribution rate, whereas this chapter does.

4.2.3 Parameter choices

The default model parameter choices of this chapter are contained in Table 4.1. The default parameters for n, m, γ, μ, λ and σ are the same as in chapters 2 and 3. The benchmark choice for the cointegration coefficient κ is chosen to be equal to 0.1. Results are shown for a variety of choices for κ, because empirical estimates are scarce and fall in a wide range. Benzoni, Collin-Dufresne, and Goldstein (2007) find an estimate for the cointegration coefficient of 0.2052 when using data going back to 1929, while the estimate is as low as 0.0475 when relying on the post-World War II sample period. Therefore, this chapter will provide a sensitivity analysis of results for alternative parameter choices, showing results for $\kappa = 0,\ 0.02,\ 0.05,\ 0.1$ and 0.2. This sensitivity analysis reveals how results depend on the horizon at which the interrelation between stock and labor markets takes place. Higher values for κ imply that cointegration between stock and labor markets takes place at a shorter horizon.

The choice for κ does not affect the long-run growth rate of labor earnings. If $\kappa > 0$, the model features a stationary dividend-earnings ratio, implying that the

[5] Thus, whereas the labor income shocks $z_{1,t}$ are temporary in nature in the presence of cointegration (see footnote 4), these shocks become permanent in the absence of cointegration.

π	0.20	contribution rate
n	40	number of working years
m	20	number of retirement years
γ	5	relative risk aversion
r	0.02	riskfree rate
κ	0.1	cointegration coefficient
μ	0.03	expected excess return on stocks
g_D	0.018	expected growth rate of dividends
σ_3	0.05	volatility of dividends
σ	0.2	volatility of stock portfolio
ν_1	0	volatility of temporary labor income shocks
ν_3	σ_3	

Table 4.1: Default model parameter values.

long-run growth rate of labor earnings coincides with the long-run growth rate g_D of dividends, and is assumed equal to 1.8% in the benchmark parameter set. The default choice $g_D = 1.8\%$ for the expected growth rate of dividends is adopted from Benzoni, Collin-Dufresne, and Goldstein (2007).

The default parameter for the volatility of dividends is $\sigma_3 = 0.05$. This implies that dividend shocks are responsible for only a part of the total variation in stock returns because $\sigma_3 = 0.05$ is smaller than $\sigma = 0.2$. This implies that there are sources of variation in stock returns other than dividend shocks, allowing the volatility of stock returns to exceed the volatility of dividends, consistent with the data. The parameter ν_3 is determined such that there is a zero contemporaneous correlation between labor income growth and stock market returns, consistent with empirical findings. It follows from equation (4.7) that this is accomplished by setting $\nu_3 = \sigma_3$. In the benchmark case, the model abstracts from temporary labor income shocks, i.e. $\nu_1 = 0$. As explained in footnote 4, temporary income shocks have only a minor effect on portfolio holdings.

4.3 Autarky

The autarky framework corresponds to the setting in which all individuals save and invest on an individual savings account.

4.3.1 Optimization problem

Individual investors save a fixed fraction π of labor earnings during their working period and use the remaining fraction $1-\pi$ for consumption. Let the financial wealth of an investor of cohort s at time t be denoted by $F_{s,t}$. Here, cohort s refers to the cohort that starts working at time s. It is assumed that individuals do not have initial wealth, i.e.

$$F_{s,s} = 0, \tag{4.8a}$$

The financial wealth process is governed by the intertemporal budget constraint:

$$dF_{s,t} = rF_{s,t}dt + \alpha_{s,t}dX_t/X_t + \pi L_t dt, \tag{4.8b}$$

for all $s \leq t \leq s+n$, where $\alpha_{s,t}$ denotes the amount invested in the risky asset by an individual in cohort s at time t. The first term on the right-hand side of equation (4.8b) denotes the riskfree return on wealth, while the second term denotes the revenues from risk taking. The third term represents savings. The wealth of cohort s accumulated at the retirement date $s+n$ is converted into a flat m-year annuity. Assuming annuities to be priced in an actuarially fair way, the terminal wealth condition is given by

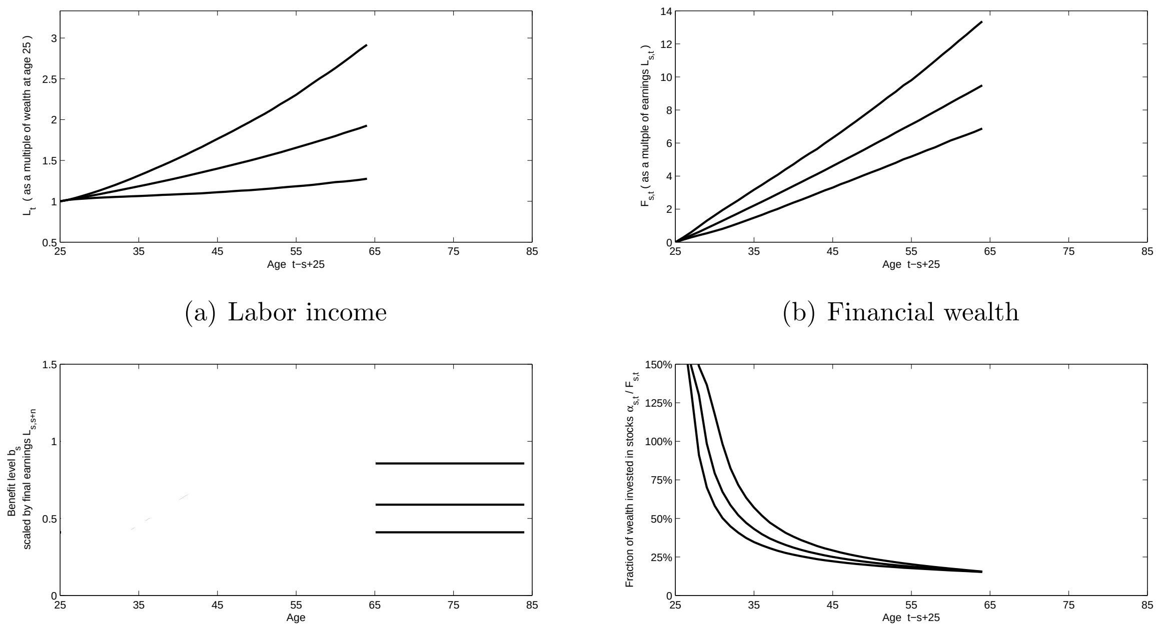
$$F_{s,s+n} = \int_0^m e^{-rv} b_{s+n} dv = \frac{b_{s+n}}{r}\left(1-e^{-rm}\right), \tag{4.8c}$$

where $F_{s,s+n}$ denotes the retirement wealth of cohort s at time $s+n$ and where the right-hand-side represents the value of an immediate annuity with payoff level b_{s+n} during the m-year retirement period.

The individual investor in autarky maximizes preferences as specified in equation (4.1) with respect to portfolio choices $\alpha(t)$ (with $s \leq t \leq s+n$), subject to the budget constraints in equations (4.8a)-(4.8c).

4.3.2 Solution

Except for a few special cases, the optimization problem cannot be solved analytically. The optimization is therefore solved numerically, using backward induction, state-space discretization, spline interpolation and Gaussian quadratures. These techniques have also been used in chapter 3 and are described in Appendix



(a) Labor income

(b) Financial wealth

(c) Benefit level

(d) Fraction $\alpha_{s,t}/F_{s,t}$ invested in stocks

Figure 4.1: *5%, 50% and 95% quantiles for a number of model variables in autarky for a cohort that enters the labor market at time s. The financial wealth levels $F_{s,t}$ in subfigure (b) are expressed in terms of the labor income level L_t. The benefit level b_s in subfigure (c) are expressed in terms of the final wage L_{s+n} of cohort s at time $s+n$. Calculations are based upon the default parameters as contained in Table 4.1. Without loss of generality, the labor income level L_s at the time of labor-market entrance is normalized to unity.*

3.A. There are three state variables: financial wealth $F_{s,t}$, labor earnings L_t and the (logarithmic) dividend-earnings ratio y_t. As explained in Benzoni, Collin–Dufresne, and Goldstein (2007), the model has a scaling feature which reduces the number of state variables from three to two: F_t/L_t and y_t.

Figure 4.1 illustrates the optimal solution strategy. Calculations are based upon the default parameters as contained in Table 4.1. Notice that financial wealth levels remain positive in practically all scenarios. Hence, imposing a borrowing constraint (as applied in chapter 2) would not affect results much.[6] Figure 4.1

[6]The presence of labor income risk and the restriction of a constant savings rate reduce the risk-bearing capacity of young investors in comparison to the model of chapters 2 and 3. Hence, a borrowing constraint would be less binding in the current chapter.

shows that the fraction $\alpha_{s,t}/F_{s,t}$ of financial wealth invested in stocks is decreasing over the life-cycle: young investors have a positive demand for stocks, similar to the result found in chapter 2 where labor income risk was absent. Hence, the default parameters do not yield the result of Benzoni, Collin-Dufresne, and Goldstein (2007) that young investors have a negative demand for stocks. Figure 4.2 examines this issue further, by analyzing how the portfolio allocation of stocks over the life-cycle is affected by the parameter choices for κ and σ_3. The figure illustrates that the demand for stocks by young investors can be both negative as well as positive, depending on the parameterization of the model. The possibility of a negative demand for stocks by young investors has been used by Benzoni, Collin-Dufresne, and Goldstein (2007) to explain low participation levels in the stock market of young individuals (the stock-market participation puzzle). This occurs if cointegration is strong: in this case young investors have a negative demand for stocks as this provides them with a (partial) hedge against shocks in future labor earnings.

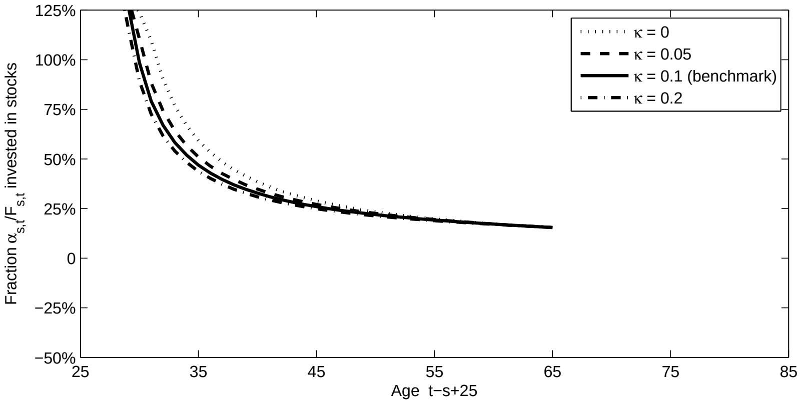
Figure 4.2 points out that the negative demand for stocks by young investors, reported in Benzoni, Collin-Dufresne, and Goldstein (2007), is thus not robust with respect to alternative parameter choices. Young investors have a negative demand for stocks only in the situation in which dividend shocks are the dominating source of variation in stock returns. Once it is recognized that stock returns are also affected by other risk sources, like mispricing, asset bubbles or time-variation in discount rates, the effect of cointegration on portfolio holdings is reduced and the demand for stocks by young investors is positive.

4.4 Risk sharing

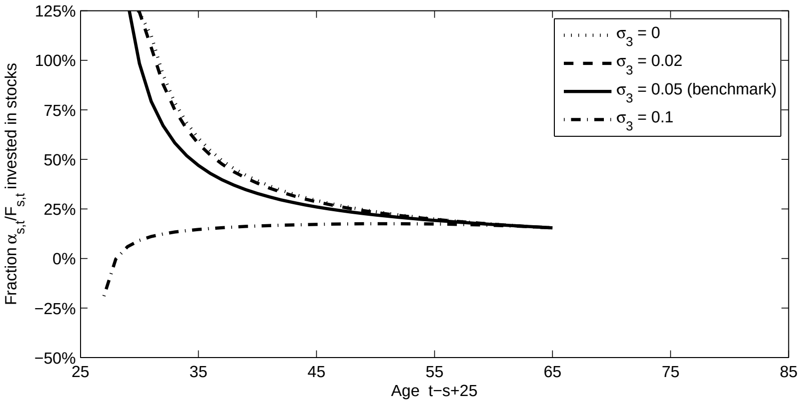
The risk-sharing case corresponds to the setting in which a social planner reallocates risk across cohorts.

4.4.1 Optimization problem

Similar to previous chapters, the social planner takes the form of a benevolent pension fund. All working cohorts pledge a fraction π of their labor earnings to the pension fund, where the savings rate π is the same as in autarky. At the



(a) Sensitivity with respect to κ



(b) Sensitivity with respect to σ_3

Figure 4.2: *The 50% quantile for the fraction $\alpha_{s,t}/F_{s,t}$ of financial wealth invested in stocks, for four different choices of the cointegration coefficient κ and the volatility of dividends σ_3. Calculations correspond to the benchmark parameters contained in Table 4.1.*

retirement date, participants receive an m-year flat annuity. The payoff received by participants retiring at time t is denoted by b_t. The intertemporal budget constraint of the pension fund is given by:

$$dF_t = \begin{cases} rF_t dt + \alpha_t dX_t/X_t + n\pi L_t dt & \text{if } t \notin \mathbb{N} \\ rF_t dt + \alpha_t dX_t/X_t + n\pi L_t dt - \frac{b_t}{r}\left(1 - e^{-rm}\right) & \text{if } t \in \mathbb{N} \end{cases} \tag{4.9}$$

where the first term on the right-hand-side represents the riskfree return on financial assets F_t. The second term on the right-hand-side represents denotes the excess return from investments in stocks, in which α_t denotes the amount invested in the stock market by the pension fund at time t. The third term on the right-hand-side denotes the (actuarially fair) price of the annuity that is provided to a retiring cohort at each discrete point in time $t \in \mathbb{N}$. The variable b_t denotes the payoff-level of the annuity that is received the cohort that retires at time t, i.e. the cohort that entered the labor market at time $t - n$.

To determine the initial wealth level of the pension fund, I follow the approach of Gollier (2008) by taking the perspective of a pension reform in which the n working cohorts in autarky agree to transfer their wealth to a social planner.[7] Before the date of the pension reform, individual investors are saving on individual retirement accounts according to the optimal autarky portfolio rule. The date of the reform is normalized to t_0. This can be done without loss of generality, because the state variables y_{t_0} and F_{t_0}/L_{t_0} both adopt a stationary distribution in autarky. The analysis is restricted to a single scenario for the reform, namely the scenario in which the values of y_{t_0} and F_{t_0}/L_{t_0} are equal to their unconditional means (i.e. their long-term averages).

Participants do not save or investment outside the pension fund. The pension fund optimizes the aggregated utility of all current workers and future cohorts:

$$\max_{\alpha_t, b_t \ \forall t > t_0} \left\{ \mathbf{E}_{t_0} \left[\sum_{t=t_0}^{\infty} \delta^{-t} U_{t-n} \right] \right\}, \tag{4.10}$$

with respect to the decisions for portfolio holdings and benefit payments, subject to the budget constraint in equation (4.9). Parameter δ represents the discount factor that the social planner uses to weigh the relative importance of the cohorts.

[7] A date of initiation is required in an evaluation of risk sharing, because otherwise the gains from risk sharing becomes infinitely large.

Following Gollier (2008), parameter δ is set by the social planner such that the welfare gain from risk sharing is proportionally equally divided among cohorts:[8]

$$\frac{CEQ_{s'}}{CEQ_s} = \frac{CEQ_{s'}^{aut}}{CEQ_s^{aut}}, \tag{4.11}$$

for all cohorts $s, s' > t_0$, where CEQ_s and CEQ_s^{aut} represent the certainty-equivalent retirement consumption level of cohort s in the pension fund and in autarky:

$$\frac{(CEQ_s)^{1-\gamma}}{1-\gamma} \equiv \mathbf{E}_{t_0}\left[\frac{(b_{s+n})^{1-\gamma}}{1-\gamma}\right] \tag{4.12a}$$

$$\frac{(CEQ_s^{aut})^{1-\gamma}}{1-\gamma} \equiv \mathbf{E}_{t_0}\left[\frac{(b_{s+n}^{aut})^{1-\gamma}}{1-\gamma}\right] \tag{4.12b}$$

for all s, where b_{s+n} and b_{s+n}^{aut} denote the benefit level in the risk-sharing case and in autarky respectively. The welfare gain associated with risk sharing can now be expressed in terms of the percentage change in CEQ_s, which is the same for all cohorts. By definition, risk sharing is Pareto-efficient in comparison to autarky, because the social planner is able to replicate the optimal individual strategy and hence is able to ensure that all cohorts are at least as well off as in autarky.

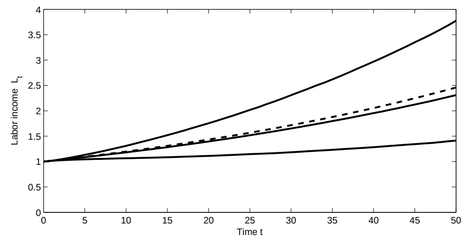
4.4.2 Solution

The problem of the social planner is solved numerically by using the same methods as in section 4.3.2.

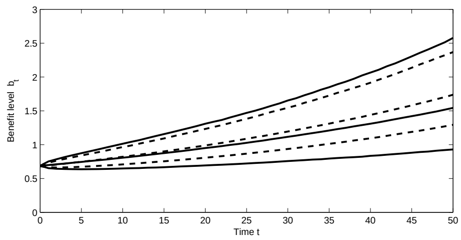
Figure 4.3 illustrates the results for the benchmark parameters. Consistent with the two-agent analysis in section 1.2.5, the demand for the risky asset is substantially reduced by the presence of labor-income risk. The economic intuition for this result is that unborn cohorts are already exposed to financial market risk via their human wealth, and therefore have a lower appetite to share in current risks via risk-sharing transfers.

Figure 4.4 illustrates how the outcomes are affected in an scenario in which there is a single negative financial shock. This illustrative example provides us with better understanding of the way in which the risk-sharing solution is affected

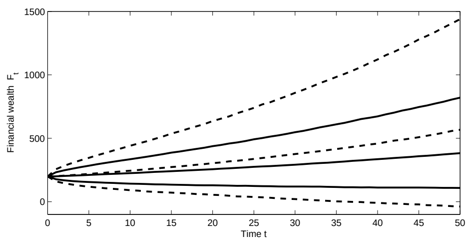
[8] In chapters 2 and 3, there was no growth in labor income, implying that $CEQ_s^{aut} = CEQ_{s'}^{aut}$ for all s,s', which causes equation (4.11) to simplify into equations (2.24) and (3.9) in chapters 2 and 3 respectively.



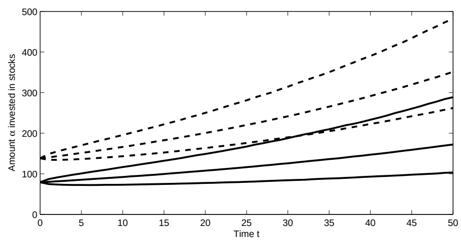
(a) Labor income



(b) Benefit level



(c) Financial wealth



(d) Amount invested in stocks

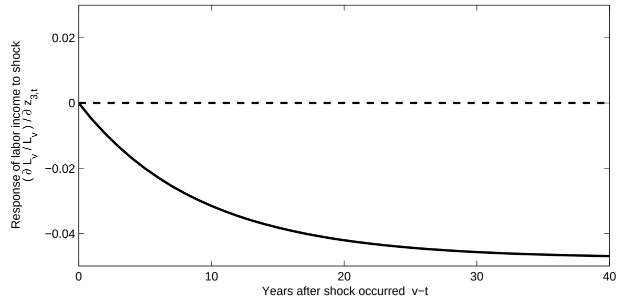
Figure 4.3: *The 5%, 50% and 95% quantiles for a number of model variables in the risk sharing solution. The solid lines represent the situation in presence of labor income risk and correspond to the default parameter values as contained in Table 4.1. The dashed lines represent the case where labor income risk is absent (i.e. the special case where $\kappa = 0$.)*

by the long-run dynamics of labor income risk. Subfigure (a) illustrates that labor income responds quite slowly to the negative financial shock (see subfigure a). The effect of the financial shock on labor earnings materializes at an horizon of roughly $1/\kappa$=10 years, where κ is the cointegration coefficient. Indeed, the Figure illustrates that roughly half of the impact on labor income materializes in the first 10 years after the shock. The financial wealth response to the economic shock is depicted in subfigure (b) and is roughly the mirror image of the labor-income response in subfigure (a). The financial shocks causes an immediate drop in the value of financial assets, but the pension fund recovers from this shock over time. The speed at which the financial position of the pension fund recovers is roughly equal to the speed of the labor income response. Subfigure (c) illustrates that all generations are proportionally equally affected in the optimal solution strategy, consistent with the consumption smoothing intuition derived in equation (1.13) in chapter 1.

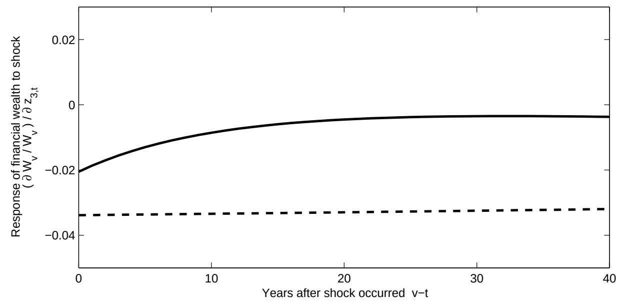
The first column in Table 4.2 reports the gain from risk sharing for various parameter choices of κ. The table shows that labor-income risk reduces the risk appetite of the pension fund and reduces the attractiveness of risk sharing. For the default parameters, the fraction of pension fund assets invested in stocks drops from 66.2% to 37.8%. The welfare gain from risk sharing reduces from 23.0% to 6.9%, a reduction of (23.0-6.9)/23.0=70%.[9] The intuition for this result is that it becomes less attractive for future generations to share in current financial risks if these risks relate positively to their future labor earnings. If stock and labor markets move together in the long run, then it becomes unattractive for future generations to share in current financial risks via the pension fund, because they are already exposed via their human wealth. As a result, the gains from risk sharing are reduced in comparison to an analysis in which labor-income risk is absent.

Table 4.2 shows that the gain from risk sharing is reduced by more than half for a wide range of parameter choices for the cointegration coefficient, varying

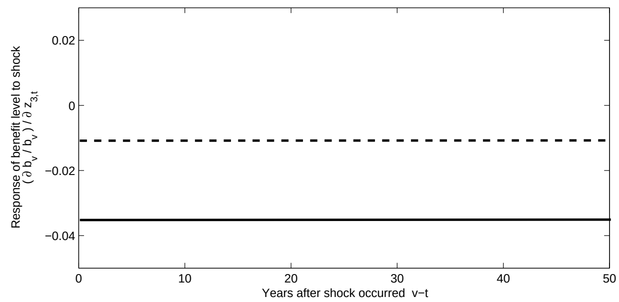
[9]Notice that the welfare gain from risk sharing in absence of labor income, reported to be 23.0%, is larger than in chapters 2 and 3. This is due to the presence of a positive real income growth rate, which is g_D=1.8% in this chapter, whereas chapters 2 and 3 do not feature real labor income growth. Due to real labor income growth, unborn generations are richer relative to currently-living generations, which increases their risk appetite and increases the gain from risk sharing.



(a) Labor income L_t



(b) Financial wealth F_t



(c) Benefit level b_t

Figure 4.4: *The elasticity of labor income L_v, financial wealth F_v and benefit levels b_v at times $v > t$ in response to an economic shock $z_{3,t}$ at time t: $(\partial b_v/b_v)/(\partial z_{3,t})$. The illustration corresponds to a one-time negative shock $z_{3,t} = -1$. Calculations are based upon the default parameters in Table 4.1.*

	Welfare gain from risk sharing	Fraction of pension fund assets invested in stocks (at time of reform)	Half-life of impact of shock on financial wealth
$\kappa = 0$ (no labor income risk)	23.0%	66.2%	∞ years
$\kappa = 0.02$	9.4%	55.1%	47 years
$\kappa = 0.05$	8.4%	43.5%	19 years
$\kappa = 0.1$ (default)	6.9%	37.8%	10 years
$\kappa = 0.2$	6.7%	37.5%	5 years

Table 4.2: *The welfare gain from risk sharing, and the fraction $\alpha(t_0)/F(t_0)$ of pension fund assets invested in stocks at the time t_0 of the reform, for various parameter choices of κ. Calculations are based upon the default parameters in Table 4.1.*

from 0.02 to 0.2. Hence, cointegration substantially reduces the attractiveness of risk sharing, regardless the choice of the cointegration coefficient κ. The intuition for this result is that the human wealth of unborn generations has a high duration and hence correlates strongly with current stock returns, regardless of whether cointegration between stock and labor markets takes place at a horizon of 5 years, 10 years or 20 years.

The final column in Table 4.2 shows that it is no longer optimal to shift risk far into the future once labor-income risk is recognized. The length of the recovery period is characterized by the half-life of the recovery process, measured as the number of years until half of the impact of a financial shock on the financial position of the pension fund has materialized. There is no analytical solution for the half-life of the recovery process. Instead, the half-life is measured on the basis of the numerical simulation of a single shock (as in Figure 4.4(b)). In the absence of labor income risk, the half-life of the recovery process is infinite. Labor-income risk causes the optimal recovery process to become relatively short. Table 4.2 suggests that the half-life in the optimal solution is somewhere between 5 and 19 years, depending on the parameterization of the model.[10] Hence, currently-living generations absorb the majority of current financial gains and losses by themselves in the optimal risk-sharing solution, instead of shifting risk onto unborn

[10]Recall that the estimates for κ found by Benzoni, Collin-Dufresne, and Goldstein (2007) range from 0.05 (on the basis of data-set from 1945 onwards) to 0.2 (on the basis of data going back as far as 1929).

generations.

The second column in Table 4.2 reports the demand for stocks of the pension fund to be positive for all parameter choices for κ. This result is due to the fact that the model allows for sources of variation in stock returns other than dividend shocks, i.e the model allows that σ_3 is smaller than σ. In the case where $\sigma_3 = \sigma$, the human capital of the pension fund is extremely "stock-like" and the pension fund takes a large short position in stocks. Allowing $\sigma_3 < \sigma$ is therefore important, because it prevents the effect of cointegration on the demand for stocks from being overstated.

4.5 Conclusion

This chapter shows that the long-run dynamics of labor income crucially determine the optimal risk-sharing solution. Once labor-income risk is recognized, a long recovery period is no longer optimal. It is no longer optimal to shift risk far into the future. The numerical results in this chapter indicate that the optimal recovery period of a pension fund is relatively short. Currently-living generations absorb the majority of current financial gains and losses by themselves in the optimal risk-sharing solution, instead of shifting risk onto unborn generations.

It will be interesting to extend the analysis in this chapter to the case in which time-variation is discount rates is explicitly taken into account. A richer specification of the pricing kernel provides us with a better understanding of the interrelation between financial and human capital returns, which crucially determines the willingness of young and future generations to share in current financial market risk.

A second interesting avenue for further research is to assess the implications of habit-formation for risk sharing. Habit formation can play an important role in retirement saving. For example, many pension funds provide wage-indexed benefits, suggesting that retirees compare their consumption to the consumption of workers (external habit-formation). Furthermore, many pension funds aim to offer benefits that are linked to final salary, suggesting that retirees compare their current consumption to their consumption in the past (internal habit-formation). Habit formation reduces the impact of cointegration on the gains from risk sharing.

Internal habit-formation implies that retirees compare their consumption to their final labor earnings. External habit-formation implies that that retirees compare their consumption to the labor earnings of workers. In both cases, the habit feature of individual preferences provides a hedge against future labor income shocks. If future labor earnings decrease (increase), then also future consumption needs decrease since this consumption is compared to a benchmark, i.e. future labor earnings, that also decreases (increases).

A third avenue for future research is an evaluation of the welfare gains from making wage-linked bonds available in financial markets. By providing a payoff that is linked to future wages, wage-linked bonds offer protection against standard-of-living risk. The results in this chapter suggest that standard-of-living risk is important. The analysis has shown that in the absence of wage-linked bonds, the investor needs to reduce stock investments (or even take a short position) as a (partial) hedge against future labor-income shocks. This is an unattractive way to insure against labor-income risk, because it deters the investor from taking advantage of the equity premium. Furthermore, the hedge against labor-income risk via stock holdings is surely imperfect. If wage-linked bonds are issued by the government, then a much more effective hedging-instrument against labor income risk becomes available in the financial market. Investors can then simply hedge against future labor income shocks via wage-linked bonds or derivatives that are based on wage-linked bonds. Thereby, the availability of wage-linked bonds allows investors to insure themselves against future labor income shocks more effectively, and without the need to reduce the exposure to stock market risk.

Conclusion and Avenues for Future Research

This dissertation contributes to the existing economic theory on risk sharing, which teaches that financial shocks should be smoothed over as many generations as possible. I show that shifting risk into the future is not optimal anymore once labor market distortions and the long-run dynamics of labor income are recognized. Current financial shocks should be levied primarily upon currently-living generations, instead of being smoothed over as many generations as possible. As a result, it can be unattractive for a pension fund to have its investment portfolio tilted heavily towards risky assets, because gains and losses are absorbed primarily by currently-living participants.

In the context of government oversight, the results can be viewed as an economic justification for solvency rules for pension funds. If it is unattractive for future generations to be exposed to current risks, then there is a role for a regulator to require pension funds to recover from their losses in a short time-period. As an independent body, regulators are able to defend the interests of unborn generations. Thereby, regulators constitute a counterbalance to the power of pension fund's boards of trustees, which may be primarily concerned with the interests of currently-living workers, employers or voters.

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